Department of Mechanics and Machines Design
OPOLE UNIVERSITY OF TECHNOLOGY

INTRODUCTION TO MECHATRONICS

SIGNAL PROCESSING

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Definition

Signals – information transmitted between elements of the system as electric impulses (continuous or discreet). Signals represents displacements, forces, velocity, …

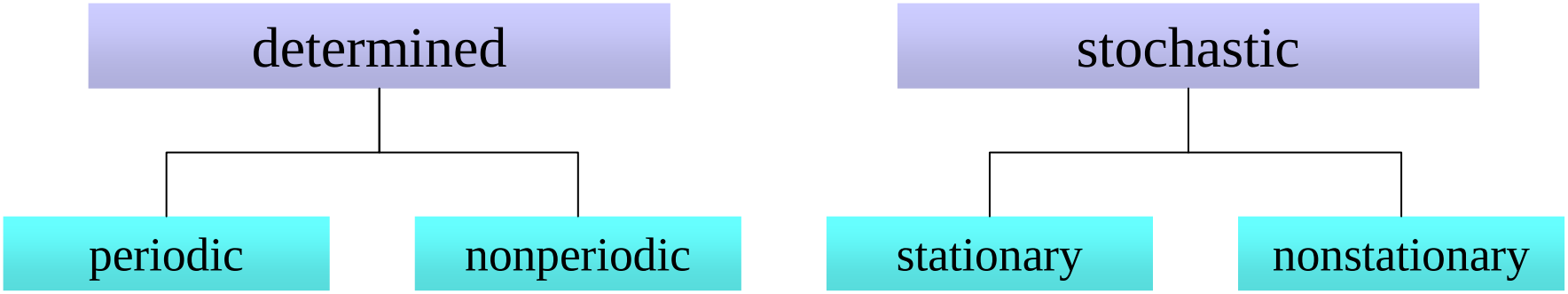
For digital processing discreet signals are needed.

Usually signals are described as time dependent function.

$\mathbf{x = x(t),} \qquad \mathbf{t_0 \leq t \leq t_1}$

From technical point of view we analyze signal registered in time range named *time of observation* $\mathbf{[0,T_{ob}]}$ (measured time).

Definition



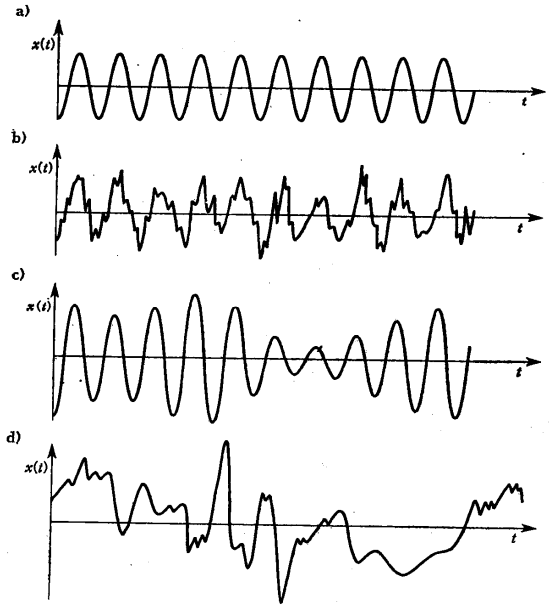
determined – described as mathematical function

$$x(t) = A\cos\sqrt{\frac{k}{m}}t$$

spring – mass system → displacement of mass

stochastic – random signals, impossible to describe by mathematical models. More exists cases of this signals are investigated as stationary signals.

Time history of the signals



To describe signal statistic parameters are determine:

ekstremum: $\hat{x} = \max[x(t)] \quad \check{x} = \min[x(t)]$

mean value: $\bar{x} = \frac{1}{T}\int_0^T x(t)dt$

variance: $\sigma_x^2 = \frac{1}{T}\int_0^T [x(t) - \bar{x}]^2 dt$

standard deviation: σ_x

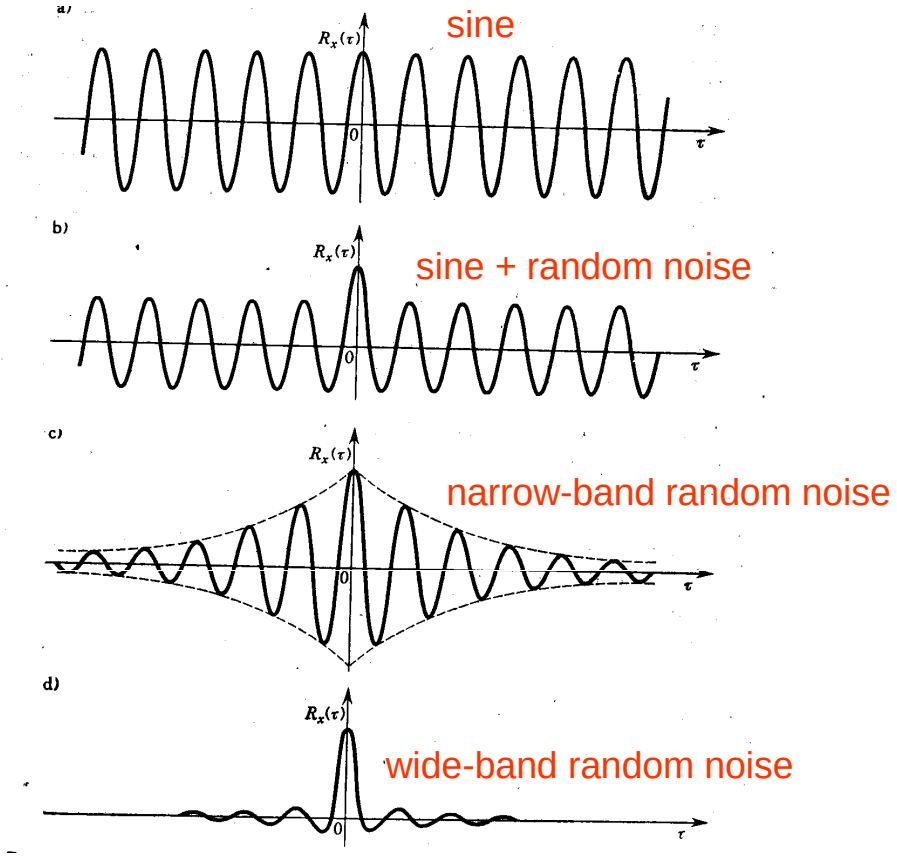
assymetry: $\gamma_x = \frac{1}{\sigma_x^3}\int_0^T [x(t) - \bar{x}]^3 dt$

kurtosis: $\beta_x = \frac{1}{\sigma_x^4}\int_0^T [x(t) - \bar{x}]^4 dt$

Characteristics of the signal

autocorrelation :

$$R_{xx} = \lim_{T\to\infty} \frac{1}{T} \int_0^T x(t)x(t+\tau)dt$$



Autocorrelation is represents the dependency of the actual value of the signal and value of the same signal shifted in time.

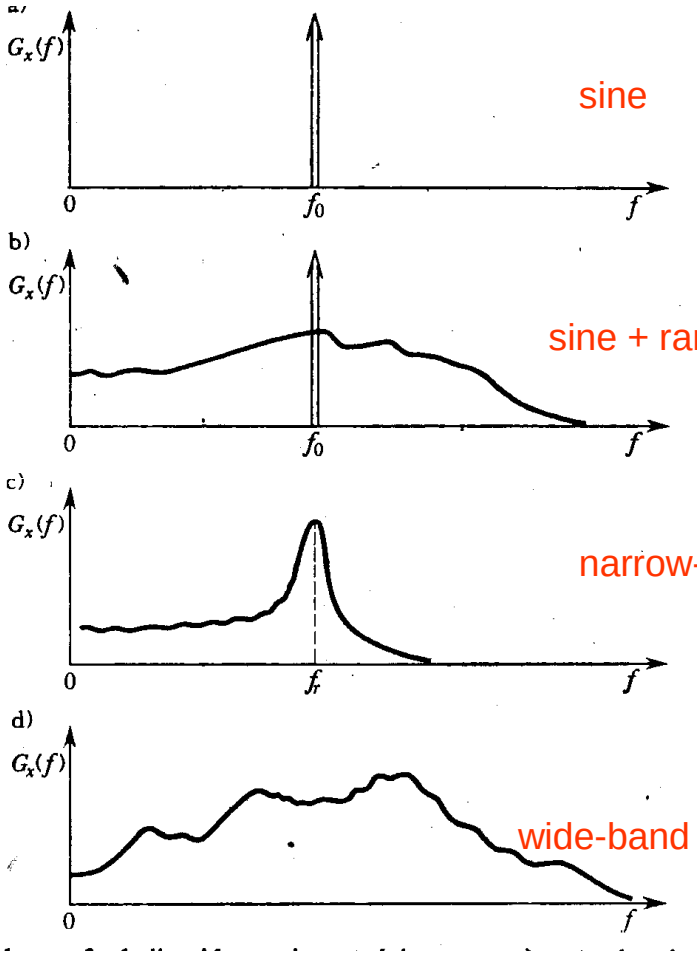
For periodic signals autocorrelation function do not change during time flow.

For random signals autocorrelation is going to zero for bigger time shifts.

Characteristics of the signal

power spectrum: $$G_{xx} = F(R_{xx}) = \int_0^{+\infty} R_{xx}(\tau) e^{-j\omega t} d\tau$$

power spectrum : describes frequencies of the components of the signals



sine

sine + random noise

narrow-band random noise

wide-band random noise

Fourier transform

- Signal can be represent as a combination of some number of the sinus and cosines functions (decomposition of the signal)
- Fourier series F(t):

$$F(t) = C_0 + \sum_{n=1}^{\infty} A_n \cos(n\omega_0 t) + \sum_{n=1}^{\infty} B_n \sin(n\omega_0 t)$$

where basic frequency

$$\omega_0 = \frac{2\pi}{T}$$

$$A_n = \frac{2}{T}\int_0^T F(t)\cos(n\omega_0 t)dt$$

$$C_0 = \frac{A_0}{2} = \frac{1}{T}\int_0^T F(t)dt$$

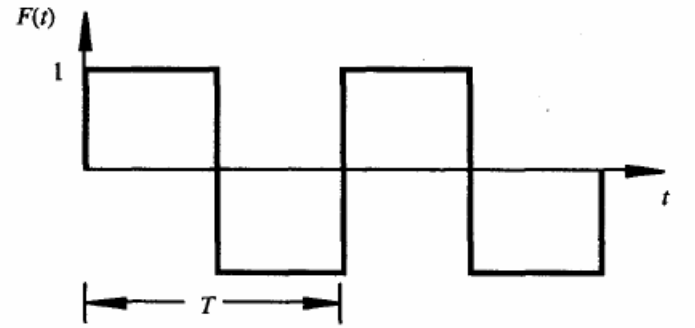
$$B_n = \frac{2}{T}\int_0^T F(t)\sin(n\omega_0 t)dt$$

$A_0 \rightarrow A_n$ for n = 0

Fourier transform

For rectangular time signal

$$F(t)=\begin{cases}1 & 0\le t\le T/2\\ -1 & T/2\le t\le T\end{cases}$$



the coefficients of the series:

$$A_0=0\text{, and }\quad A_n=0$$

$$B_n=\frac{2}{T}\left(\int_0^{T/2}\sin(n\omega_0 t)dt-\int_{T/2}^{T}\sin(n\omega_0 t)dt\right)$$

$$=\frac{2}{T}\left(-\frac{1}{n\omega_0}\left[\cos(n\omega_0 t)\right]_0^{T/2}+\frac{1}{n\omega_0}\left[\cos(n\omega_0 t)\right]_{T/2}^{T}\right)$$

$$=\frac{1}{n\pi}\left\{-\cos(n\pi)+1+1-\cos(n\pi)\right\}=\frac{2}{n\pi}\left\{1-\cos(n\pi)\right\}$$

$$=\begin{cases}\dfrac{4}{\pi} & \text{for n = 1, 3, 5 …}\\ 0 & \text{for n = 2, 4, 6 …}\end{cases}$$

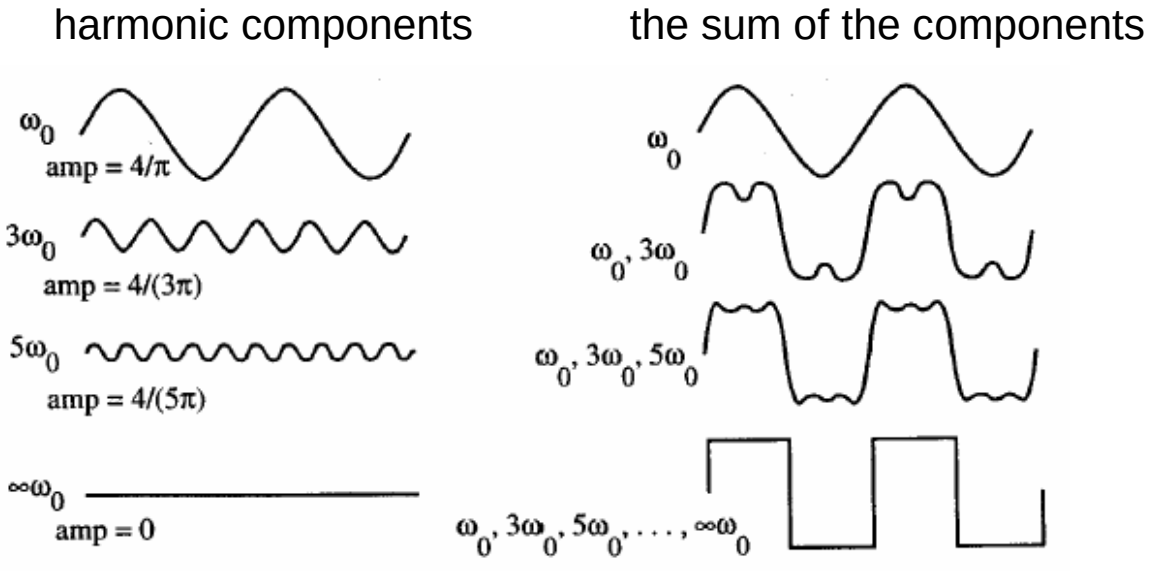
Fourier transform

Fourier series for amplitude A=1

$$F(t) = \frac{4}{\pi}\sin(\omega_0 t) + \frac{4}{3\pi}\sin(3\omega_0 t) + \frac{4}{5\pi}\sin(5\omega_0 t) + \cdots$$

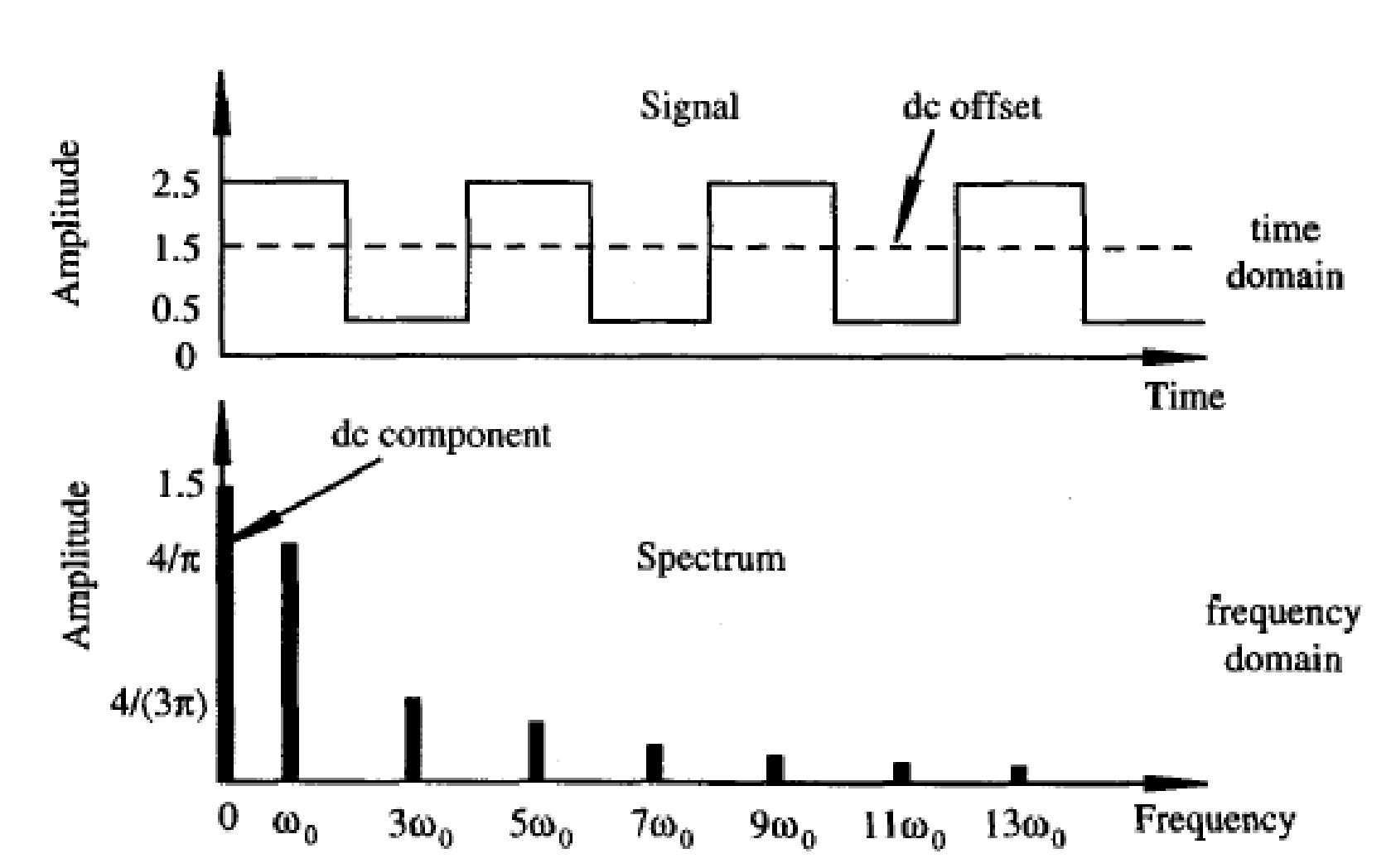
$$= \sum_{n=1}^{\infty} \frac{4}{(2n-1)\pi}\sin[(2n-1)\omega_0 t]$$

- for higher frequencies amplitude of the component is lower
- more components leads to higher accuracy of signal representation



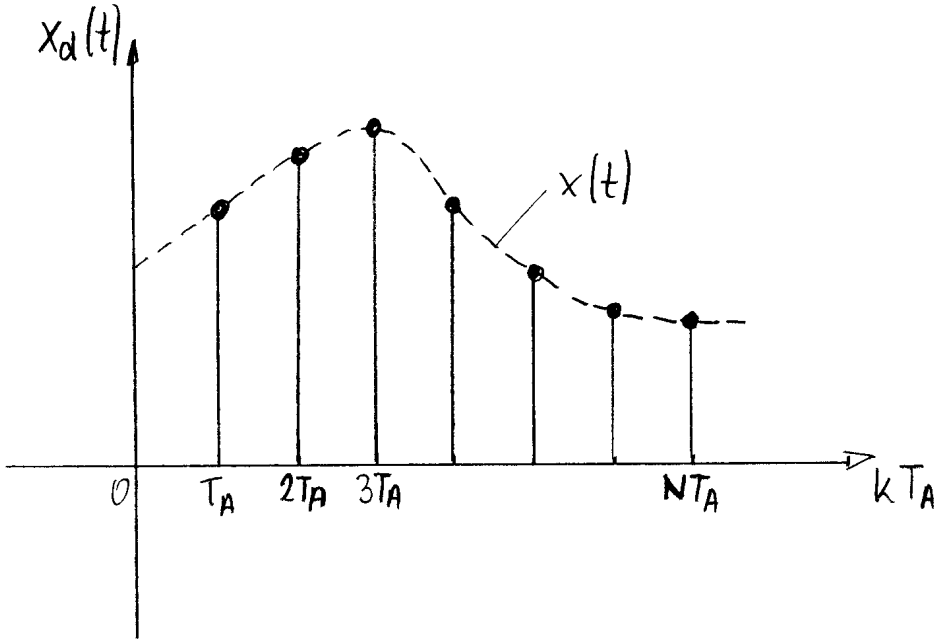
Fourier transform

Representation of the signal in time and frequency domain.



Digitization

Continuous signal have to be read as discreet and described by actual value and time moment for this value.



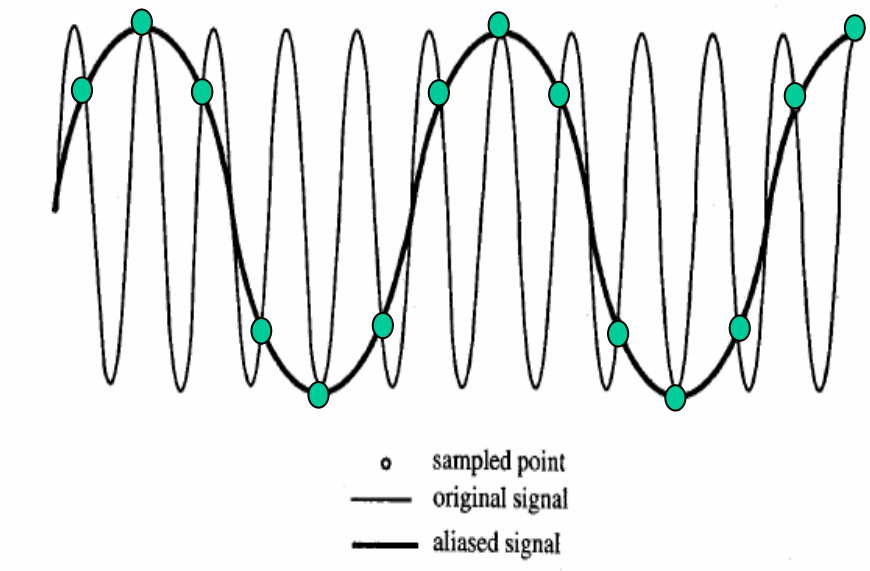
Sampling time: time history of the signal mst be readed in equal time period T_A (eq. 1 ms)

sampling frequency: $f_A = 1/T_A$ – depends on the speed of the processing system (computer – read the sample, calculations, write to the disk).

Aliasing

Real-world signals are continuous things. To represent these signals in your computer, the measurement device has to check the level of the signal and assign that level a discrete number that the computer will accept; this is called an analog-to-digital conversion. The computer then sort of "connects the dots" and, hopefully, gives you something that looks similar to the real-world signal (that's why we say it represents the signal).

The sampling rate of a system simply reflects how often an analog-to-digital conversion (ADC) takes place. Each data point on the sampled signal is represents one analog-to-digital conversion.



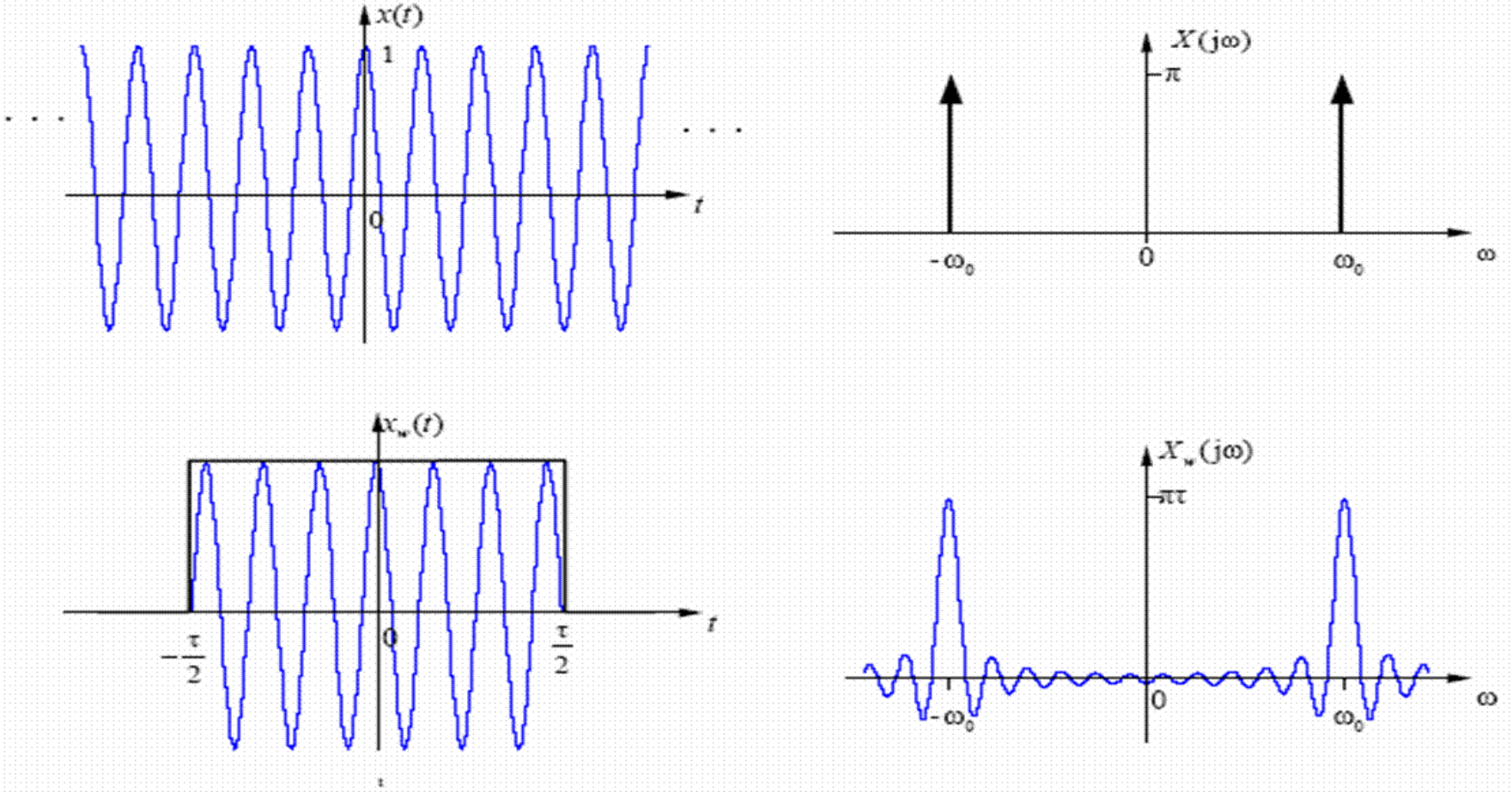
To avoid aliasing effect the sampling rate f_A must satisfy an equation:

$$f_A > 2f_{max}$$

where f_{max} is maximum frequency of the components (in frequency domain) which is exists in signal.

Leakage and windows

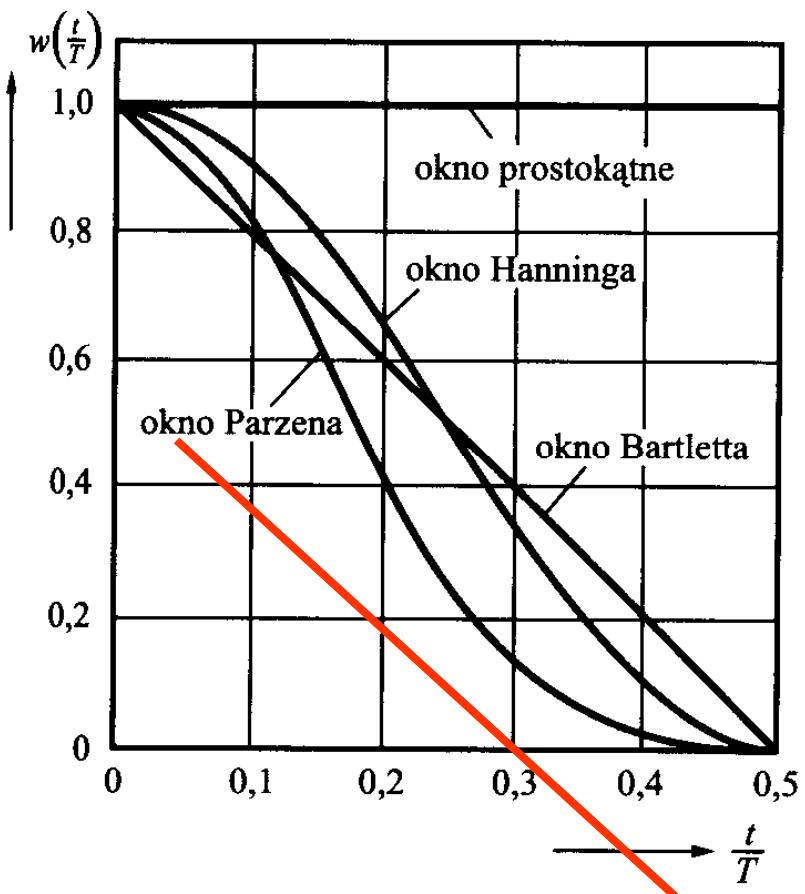
Leakage erros – occurs where finite section of continous signal is analysed.

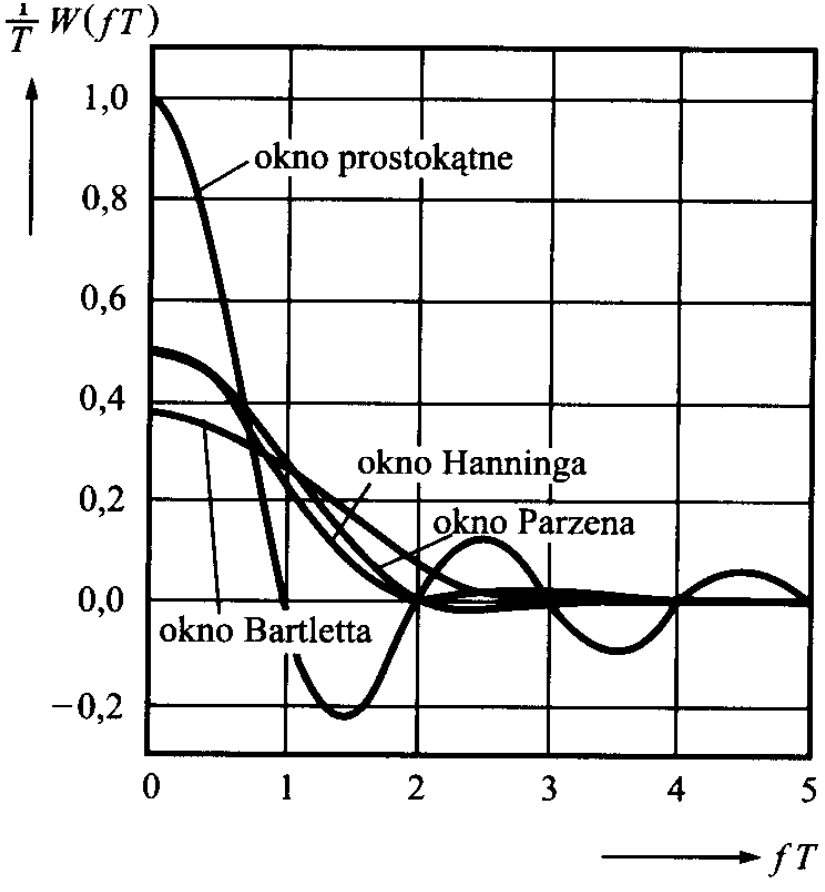


Leakage and windows

„Time window” – passes actual value of the signal x(t) with the use of same weigth function w(t):

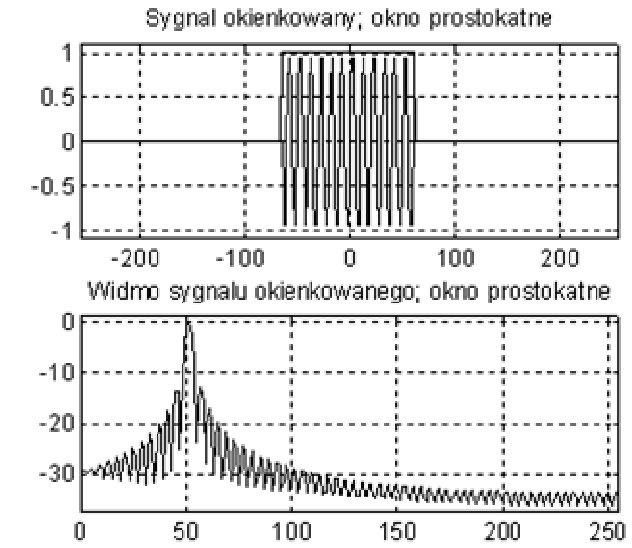
$$x_w(t) = x(t) \cdot w(t)$$

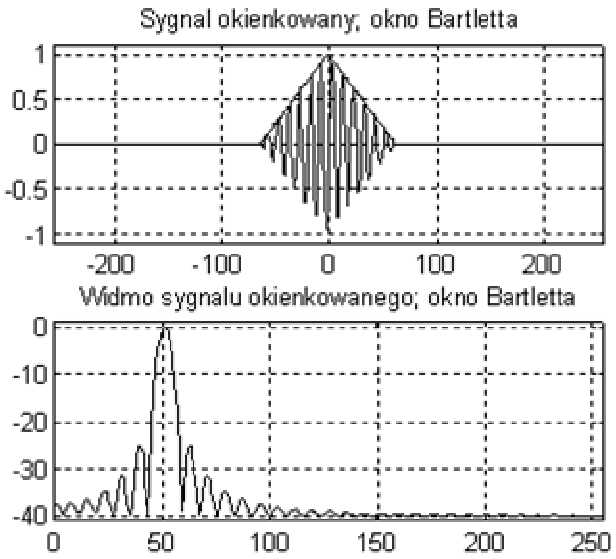


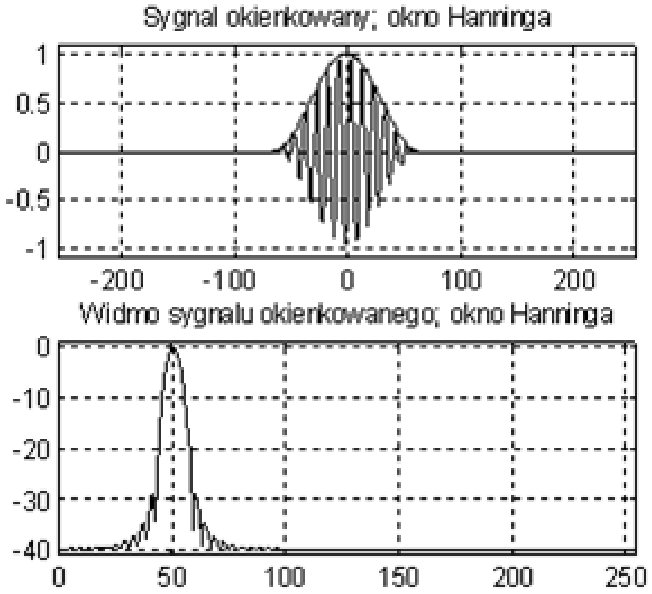


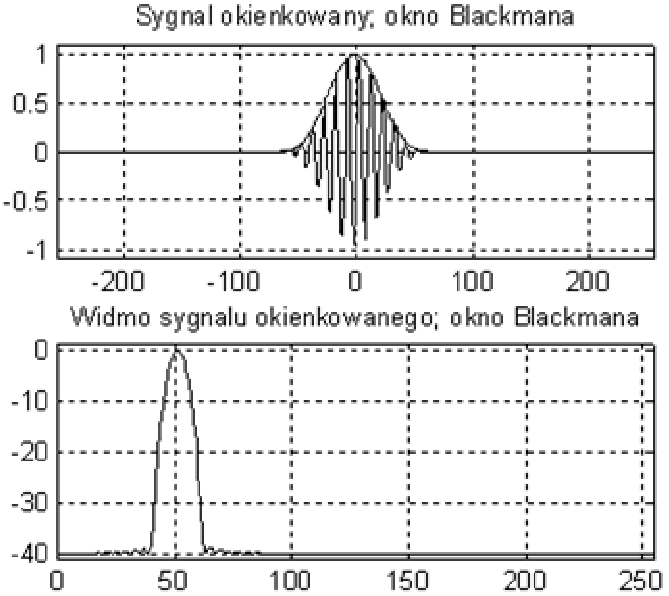
Lesson of polish language: ***okno → window***

Leakage and windows

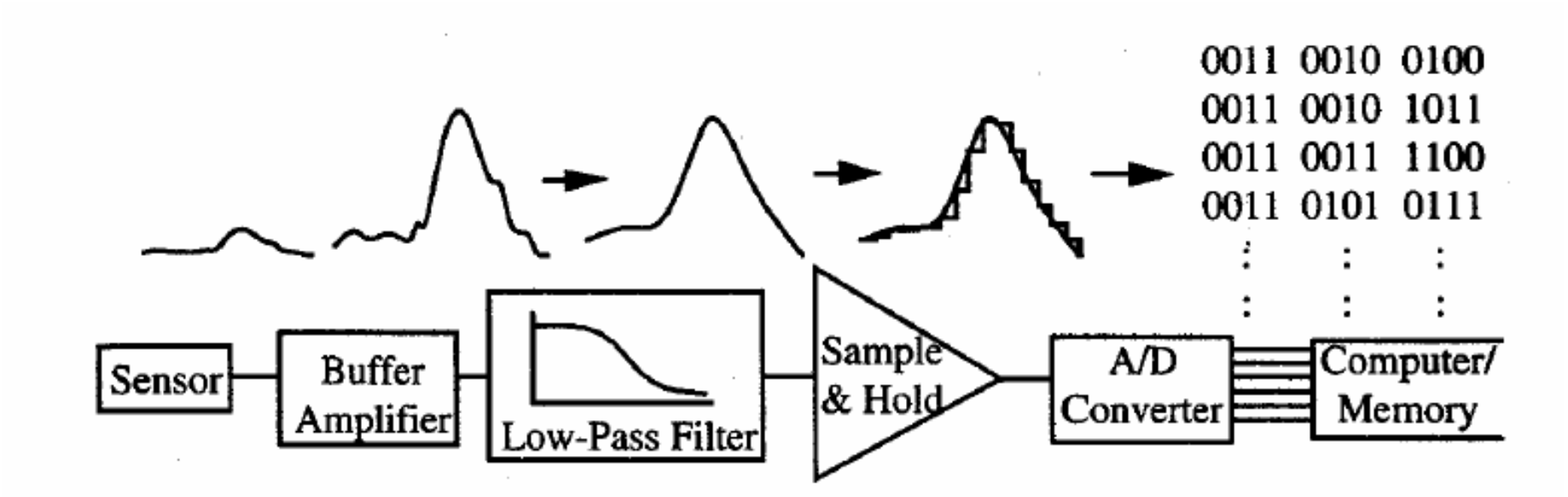


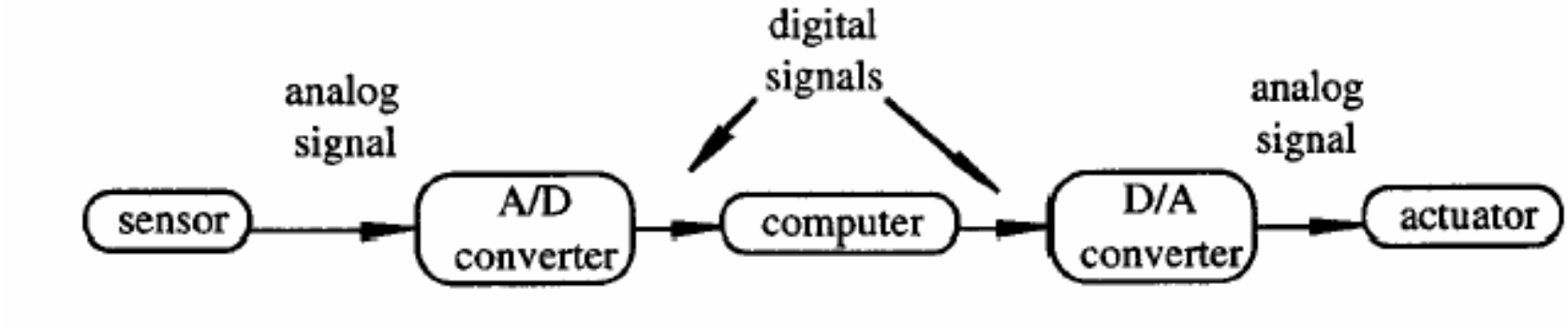






Analog – digital , digital – analog conversion





Basic structure of measurement system

Transducer: sensor, which is changing the primary physical signal to the electric one.

Processor: digital system for filtering and amplifying

Recorder: for recording and data display

Digital thermometer