Department of Mechanics and Machine Design

Graphical Programming

ODE

Ordinary differentia equations

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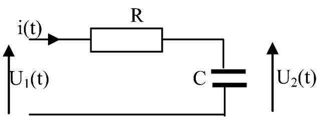
Mathematical model of the dynamic phenomena

The general form of linear differential equation with constant coefficients:

$$a_n \frac{d^n y}{dt^n} + a_{n-1} \frac{d^{n-1} y}{dt^{n-1}} + \ldots + a_1 \frac{dy}{dt} + a_0 y = b_m \frac{d^m u}{dt^m} + b_{m-1} \frac{d^{m-1} u}{dt^{m-1}} + \ldots + b_1 \frac{du}{dt} + b_0 u$$

Where to get the equations?

1. From the formulas describing the physics of the system:

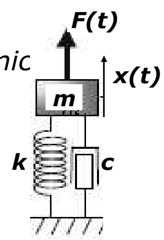


The second principle of dynamic (Newton's law)

Kirchoff's law

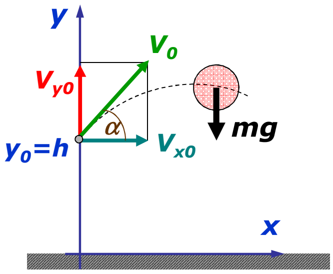
$\boldsymbol{U_1(t)=i(t)\cdot R+ U_2(t)}$

$$m \frac{d^2 x}{dt^2} + c \frac{dx}{dt} + kx = F(t)$$



Solving ODE

To solve differential equations we need to integrate them:



II principle of dynamic:

$$\vec{F} = m \cdot \vec{a}$$

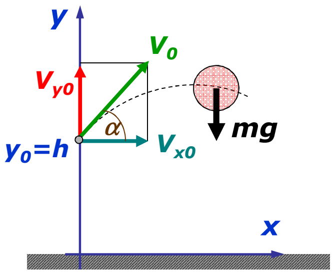
$$a = \frac{dv}{dt} \quad v = \frac{ds}{dt}$$

$$a_x = \frac{dv_x}{dt} \quad a_y = \frac{dv_y}{dt}$$

$$v_x = \frac{dx}{dt} \quad v_y = \frac{dy}{dt}$$

Solving ODE

Example:



$$F_x = 0 \quad a_x = 0 \quad \frac{dv_x}{dt} = 0$$

$$v_x(t) = \int \frac{dv_x}{dt}(t)dt = \int 0 dt = C$$

$$dla\ t = 0, v_x = v_{x0} = v_0 \cos\alpha_0 \Rightarrow C = v_{x0}$$

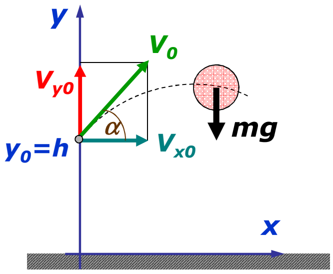
$$x(t) = \int \frac{dx}{dt}(t)dt = \int v_x(t)dt = \int v_{x0}dt = v_{x0}t + D$$

$$dla\ t = 0, x(0) = 0 \Rightarrow D = 0$$

$$x(t) = v_{x0} \cdot t$$

Solving ODE

Example:



$$F_y = -mg \qquad a_y = -g \qquad \frac{dv_y}{dt} = 0$$

$$v_y(t) = \int \frac{dv_y}{dt}(t)dt = \int -gdt = -gt + C$$

$$dla\ t = 0, v_y = v_{y0} = v_0 \sin\alpha_0 \Rightarrow C = v_{y0}$$

$$v_y(t) = -gt + v_{y0}$$

$$y(t) = \int \frac{dy}{dt}(t)dt = \int v_y dt = \int(-gt + v_{y0})dt =$$

$$= -\frac{1}{2}gt^2 + v_{y0}t + D$$

$$dla\ t = 0, y(0) = y_O \Rightarrow D = y_0 = h$$

$$y(t) = h + v_{y0}t - \frac{1}{2}gt^2$$

Solving ODE

Operating scheme method

The method consists in transforming the equation into a form in which the member of the highest derivative is on the left side of the equation and then using the integration operation.

For simplicity, the equation has the form:

$$a_n \frac{d^n y}{dt^n} + a_{n-1}\frac{d^{n-1}y}{dt^{n-1}} + \ldots + a_1 \frac{dy}{dt} + a_0 y = ku$$

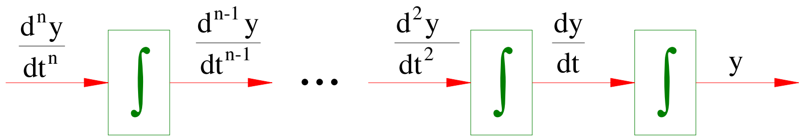
Solving ODE

Operating scheme method:

1. Transforming the equation equation due to the highest order derivative

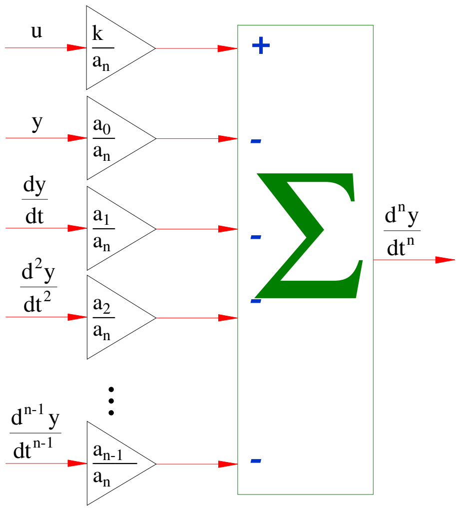
$$\boxed{\frac{d^n y}{dt^n}} = \frac{k}{a_n} u - \frac{a_{n-1}}{a_n}\frac{d^{n-1} y}{dt^{n-1}} - \ldots - \frac{a_2}{a_n}\frac{d^2 y}{dt^2} - \frac{a_1}{a_n}\frac{dy}{dt} - \frac{a_0}{a_n} y$$

2. Performing integration of subsequent derivatives



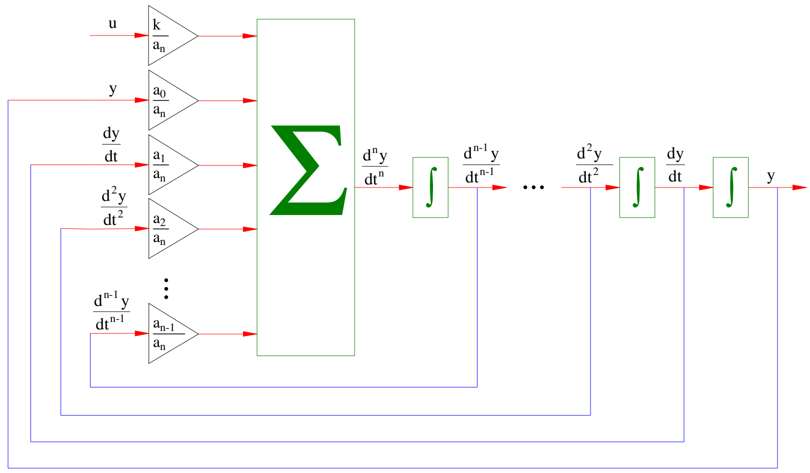
Solving ODE

3. Building an additive node for input signals for $\frac{d^n y}{dt^n}$



Solving ODE

4. Sending the integration results to the entrance of the sum node



Example

VALVE

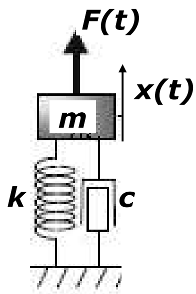
F – spring force

m – moving mass

k – spring constant

c – damping

x – displacement



$$m\frac{d^2x}{dt^2}+c\frac{dx}{dt}+kx=F(t)$$

Equation transformed due to second derivative of x:

$$\frac{d^2x}{dt^2}=\frac{1}{m}F(t)-\frac{c}{m}\frac{dx}{dt}-\frac{k}{m}x$$

Example

$$\frac{d^2x}{dt^2} = \frac{1}{m}F(t) - \frac{c}{m}\frac{dx}{dt} - \frac{k}{m}x$$