

Cyfrowe Przetwarzanie Obrazów

Digital Image Processing

Wykład 2

Lecture No 2 *



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* some part of the document has been translated, R.Pawliczek



Poitrn transformation

- convert images / images to other images

$$I \Rightarrow J \quad I \times I \times \dots \times I \Rightarrow J$$

- operations on individual pixels of the image

$$I(x,y) \Rightarrow J(x,y)$$

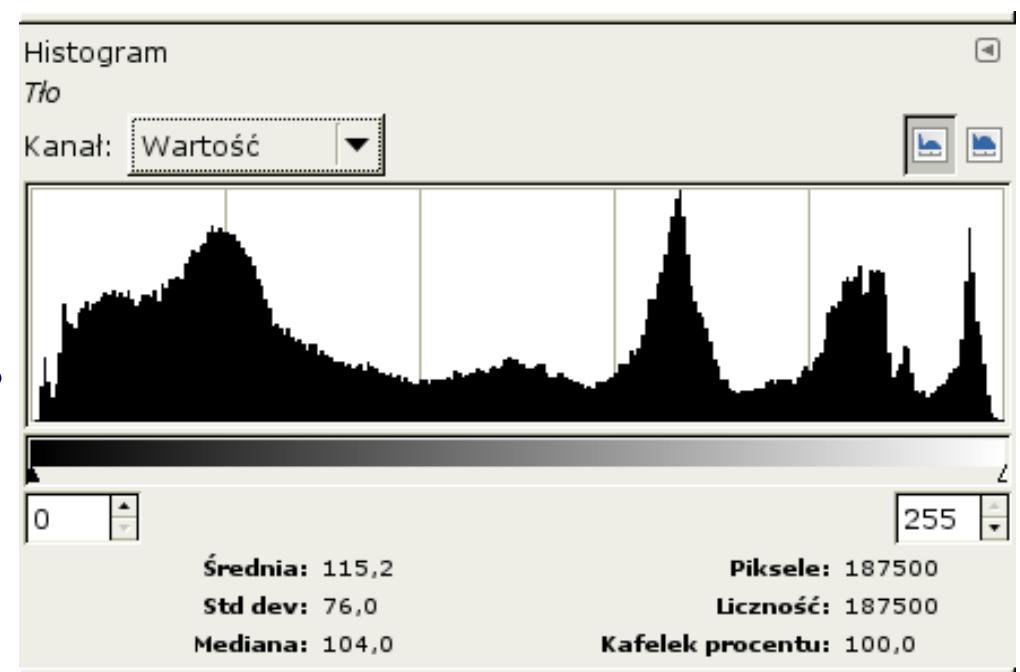
- the environment of the processed pixel has no effect on the operation
- all pixels of the same intensity are treated identically
- they serve different purposes:
 - improvement of image quality (contrast, brightness)
 - mining, emphasizing certain features
 - histogram change
 - change of colors
 - arithmetic in images (addition, subtraction)
 - Gamma correction



Histogram of a digital image

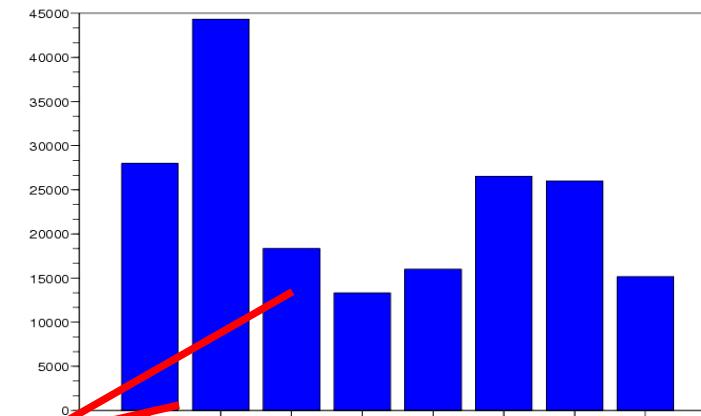
- graph showing **how many pixels of each intensity** is in the image
- all possible intensities in the image are put in the OX axis
- the Y position of the point (height of the bar) is proportional to the number of pixels X-intensity image.

$$H(i) = \text{card}\{(x, y) : \text{Image}(x, y) = i\}$$





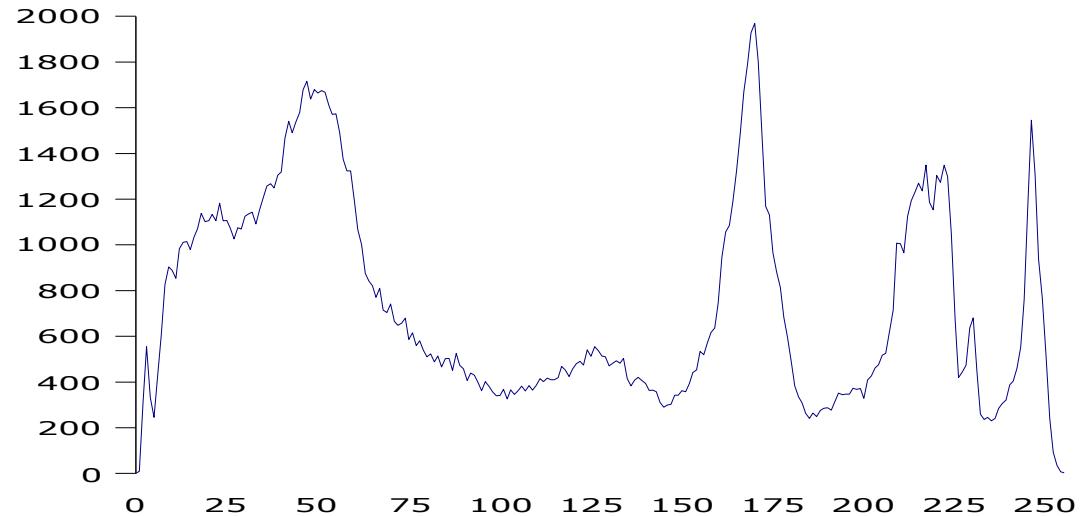
picture with 8 levels of intensity





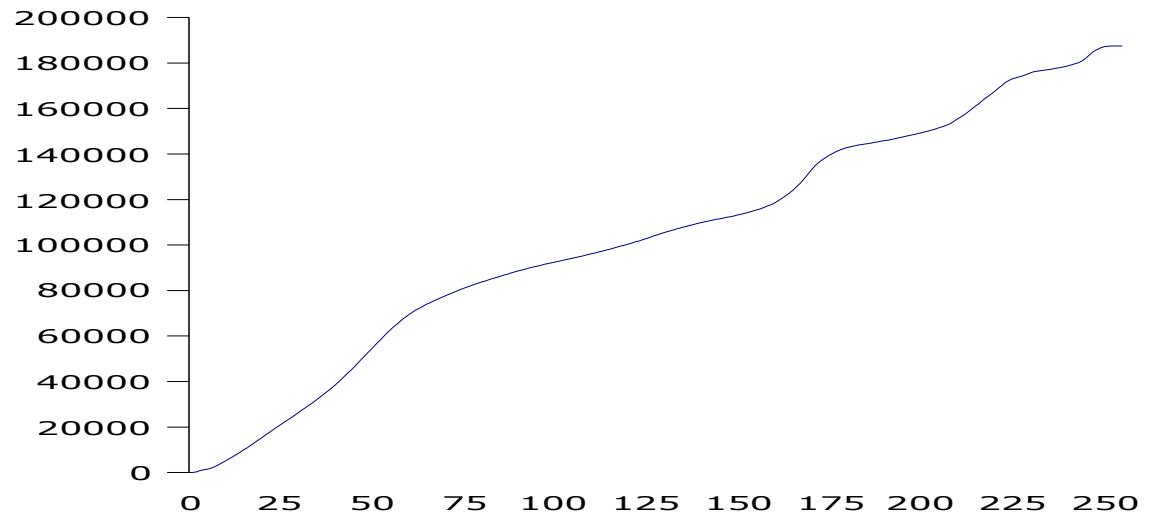
Histogram calculation (Scilab)

```
image=gray_imread('W2.jpg');
[h, w]=size(image);
hist=zeros(256,1);
for y=1:h
    for x=1:w
        lev=int(image(y,x)*255+0.5);
        hist(lev+1)=hist(lev+1)+1;
    end
end
bar(hist);
xbasc;
plot(hist);
```



cumulative histogram

```
histcum=hist;
for i=2:size(histcum,1)
    histcum(i)=histcum(i)+histcum(i-1);
end
plot(histcum);
```

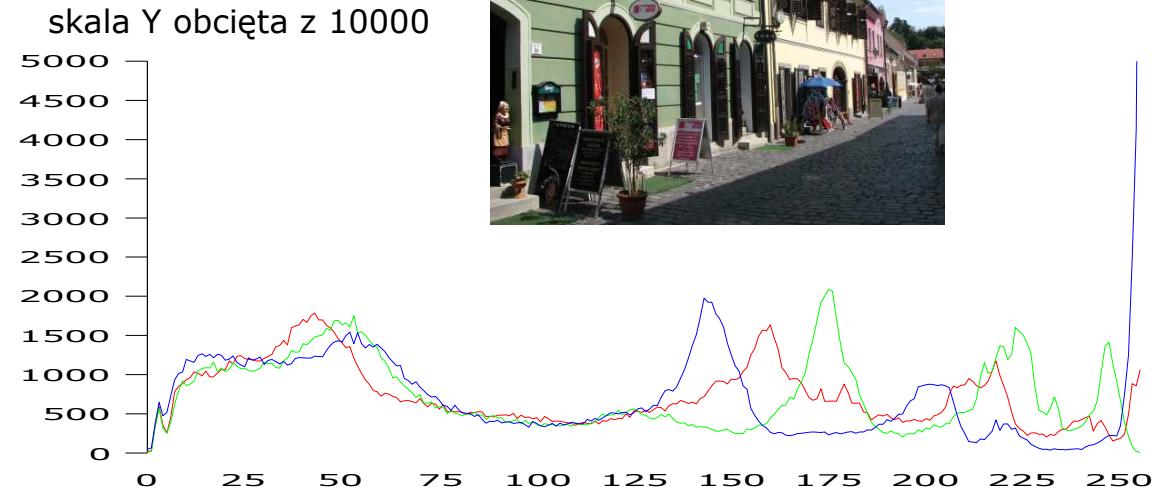




Histogram of colored images

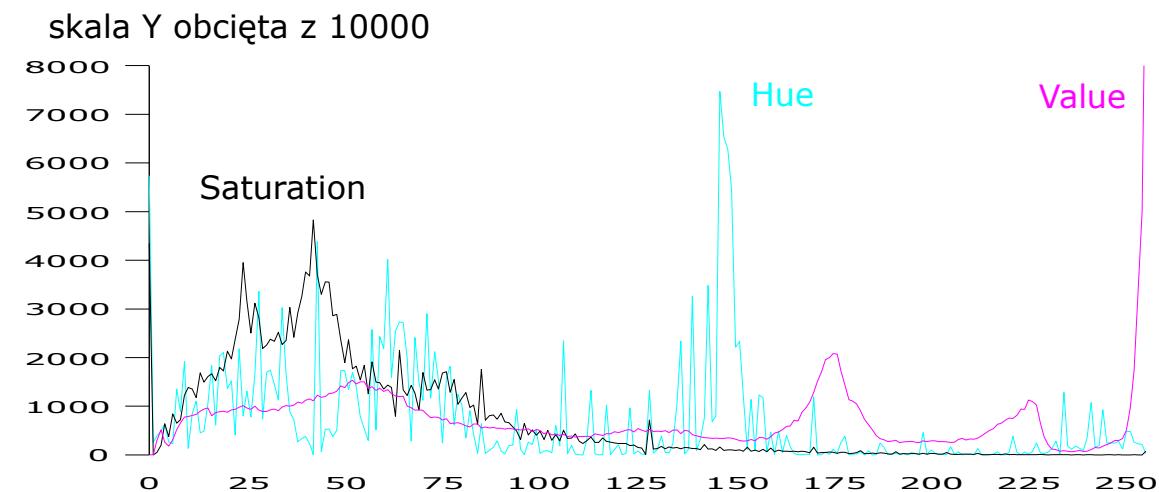
each of the RGB channels separately:

```
imRGB=imread('W2-RGB.jpg');
[h, w]=size(imRGB);
hRGB=zeros(256,3);
for y=1:h
    for x=1:w
        for k=1:3
            l=int(imRGB(y,x,k)*255+0.5);
            hRGB(l+1,k)=hRGB(l+1,k)+1;
        end
    end
end
plot(hRGB);
```



for HSV model of colors:

```
imHSV=rgb2HSV(imRGB);
hHSV=zeros(256,3);
for y=1:h
    for x=1:w
        for k=1:3
            l=int(imHSV(y,x,k)*255+0.5);
            if l<0 then; l=0; end
            hHSV(l+1,k)=hHSV(l+1,k)+1;
        end
    end
end
plot(hHSV);
```





Changing the pixel value

Look-up Table (LUT)

defines context-free operations

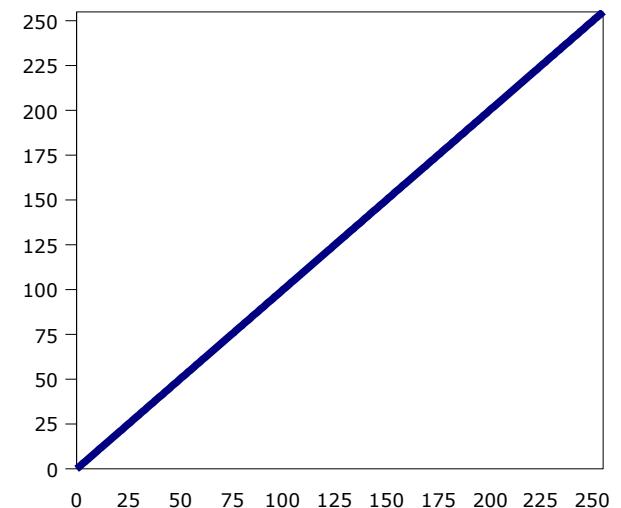
- array specifies the value of the pixel after the operation

$$I(x,y) \Rightarrow \text{LUT}(I(x,y))$$

example of LUT

piksel	f(piksel)
0	13
1	34
2	234
...	...
254	11
255	255

identity transformation LUT
input=output

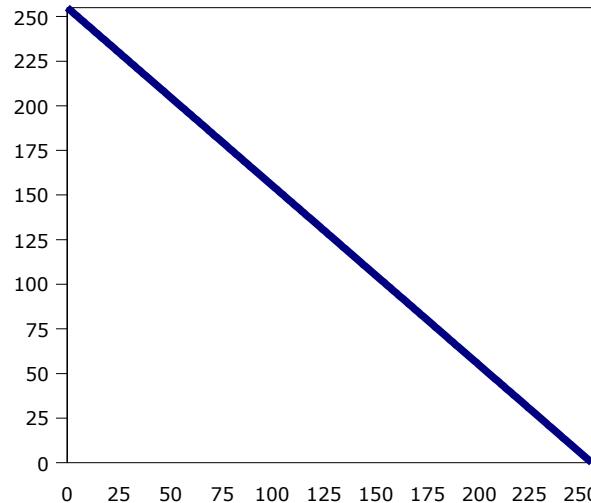


```
function res=LUTgray(image, LUT)
[h, w]=size(image);
res=zeros(h,w);
for y=1:h
    for x=1:w
        res(y,x)=LUT(int(image(y,x)*255+0.5)+1)/255;
    end
end
endfunction
```

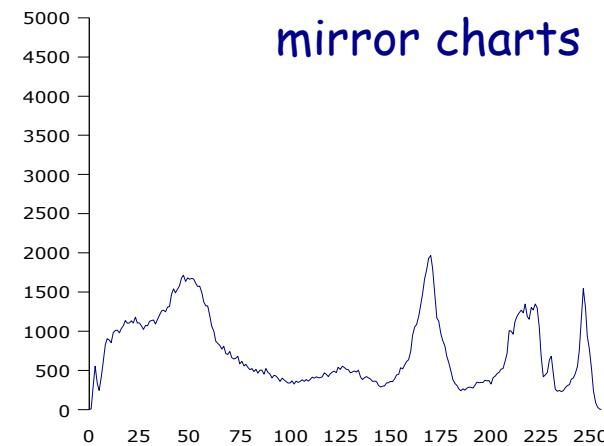


Negative image

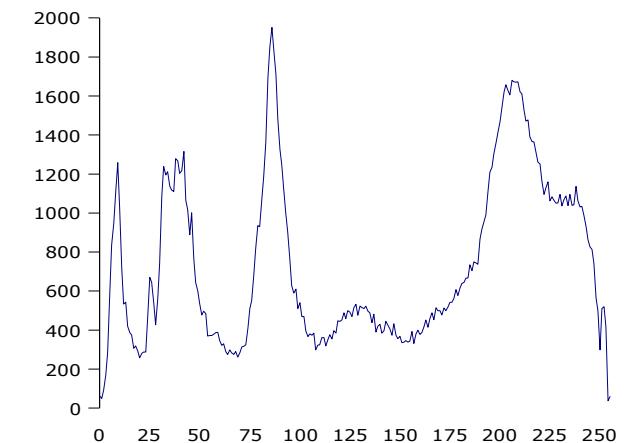
```
for i=0:255  
    LUT(i+1)=255-i;  
end  
image2=LUTgray(image,LUT) ;  
imshow(image2) ;
```



skala Y obcięta



mirror charts





Thresholding

$$Image(x, y) \Rightarrow \begin{cases} 0 & \text{if } Image(x, y) < \text{threshold} \\ 255 & \text{if } Image(x, y) \geq \text{threshold} \end{cases}$$

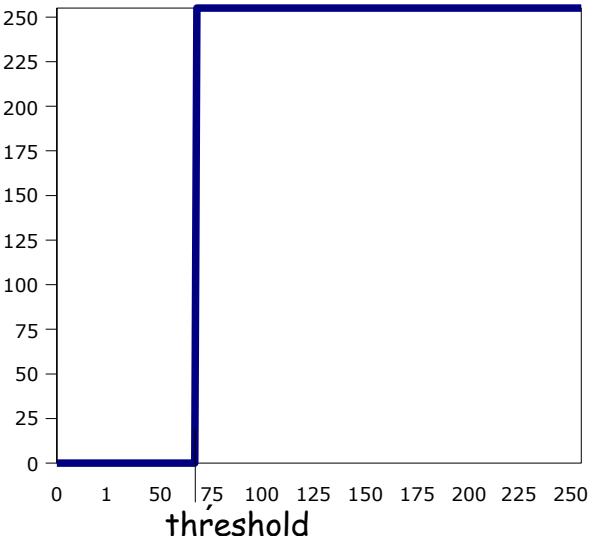
```
LUT=zeros(256,1);  
LUT(próg:256)=1;  
image2=LUTgray(image,LUT);  
imshow(image2);
```



threshold=32



threshold=127



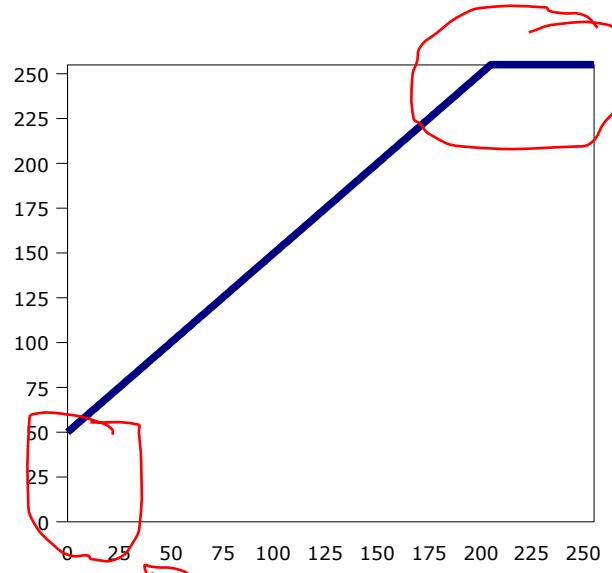
threshold=184



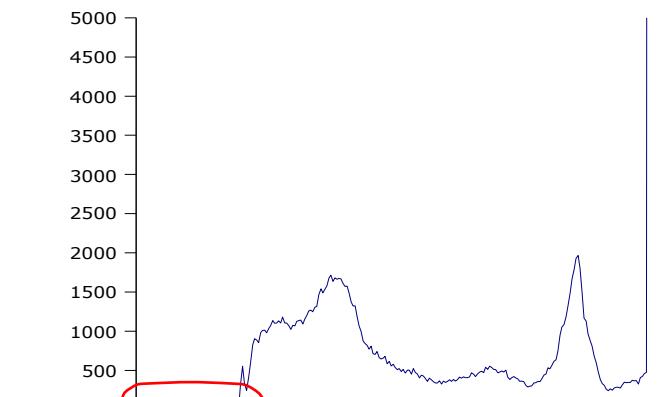
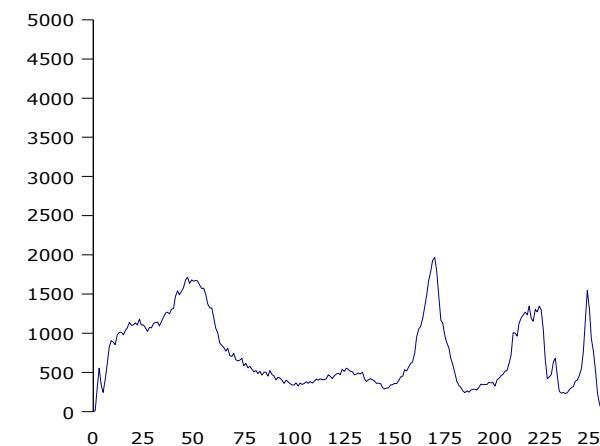


Increase brightness

```
//Increase brightness +50  
for i=0:255  
    LUT(i+1)=min(255,i+50);  
end  
image2=LUTgray(image,LUT);  
imshow(image2);
```



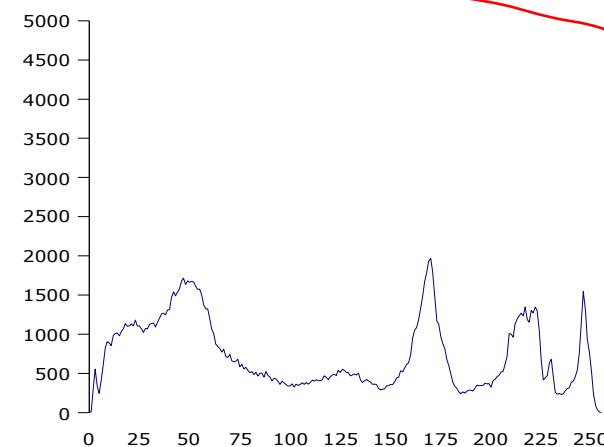
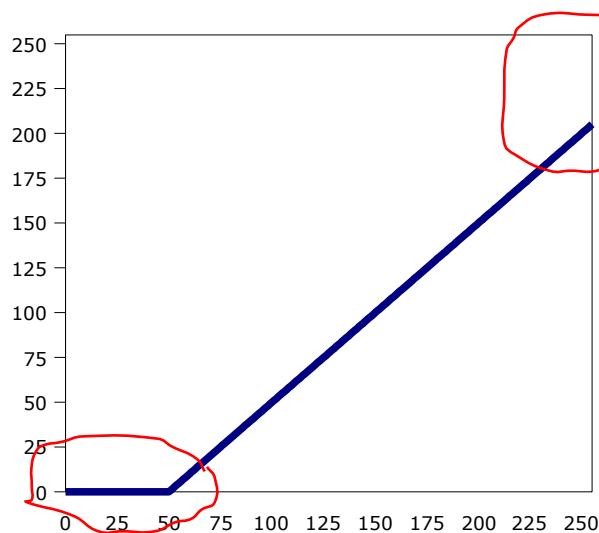
skala Y obcięta



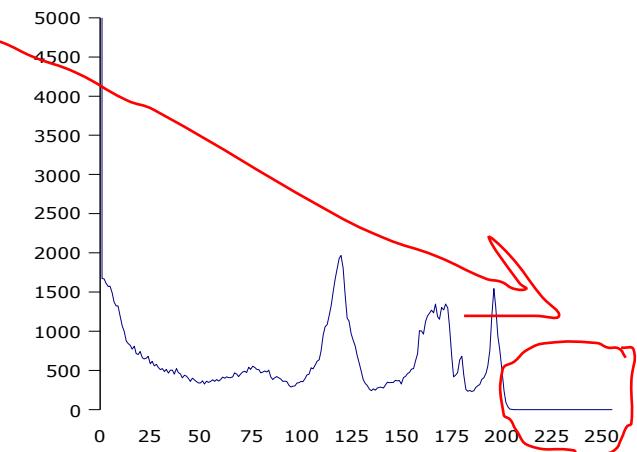


Decrease brightness

```
//Decrease brightness -50  
for i=0:255  
    LUT(i+1)=max(0,i-50);  
end  
image2=LUTgray(image,LUT);  
imshow(image2);
```



skala Y obcięta





Loss of information when changing brightness

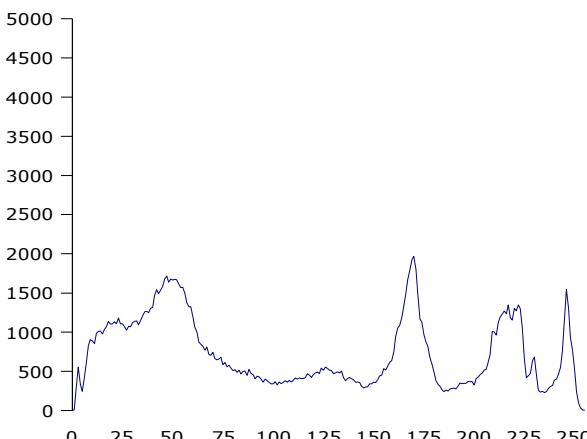
brightness -100



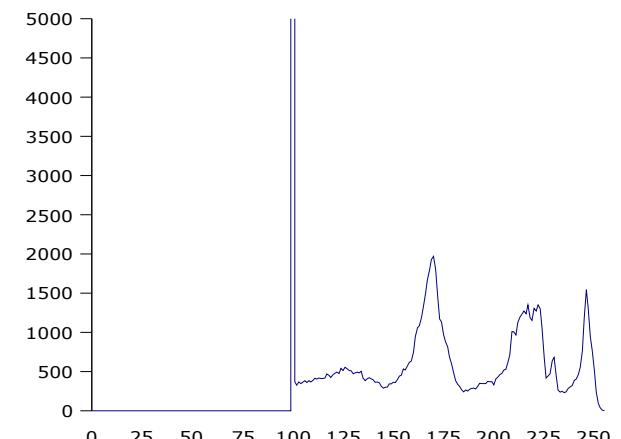
brightness +100



skala Y obcięta



- all dark intensities are cut off
- levels > 100 have not changed





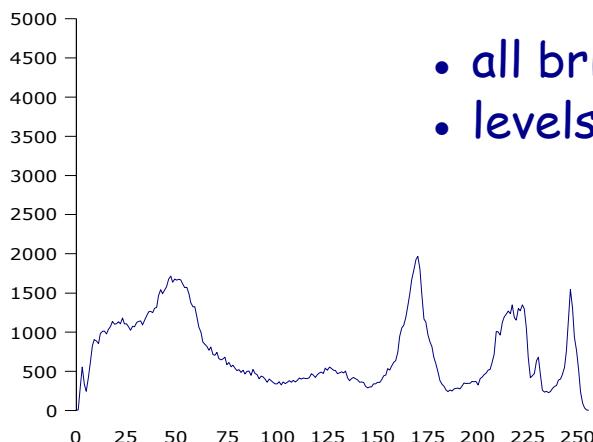
brightness +100



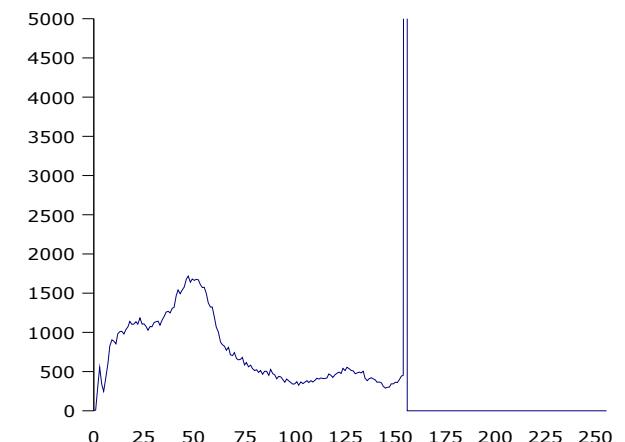
brightness -100



- all bright intensities have been cut off
- levels <255-100 have not changed



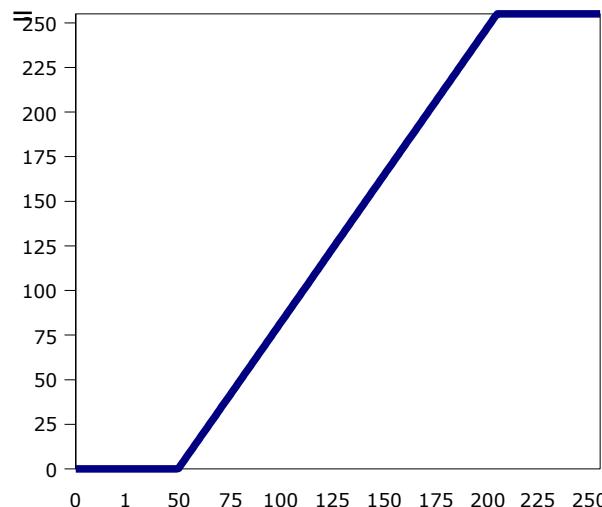
skala Y obcięta



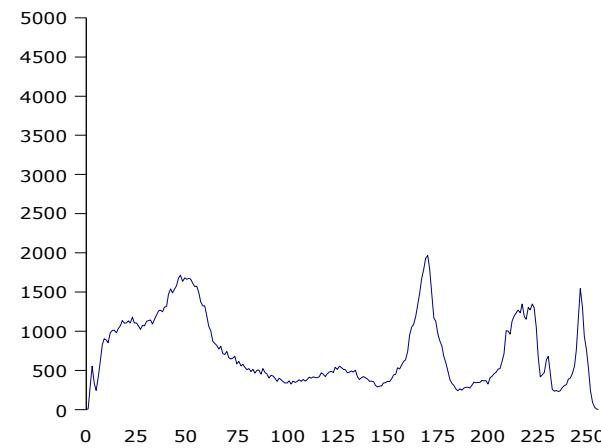


Increasing the contrast

```
// Increase contrast +50  
for i=0:255  
    LUT(i+1)=int(127+(i-127)*255/155+0.5);  
    LUT(i+1)=min(255,max(0,LUT(i+1)));  
end  
image2=LUTgray(image,LUT);  
imshow(image2);
```



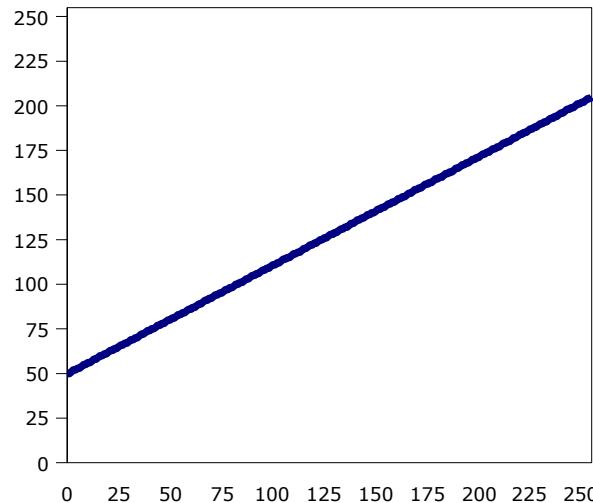
skala Y obcięta



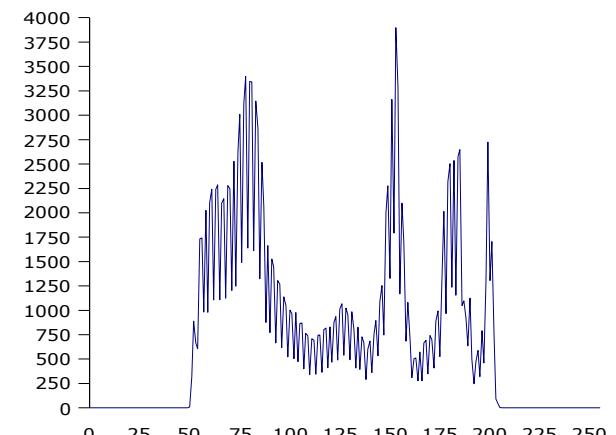
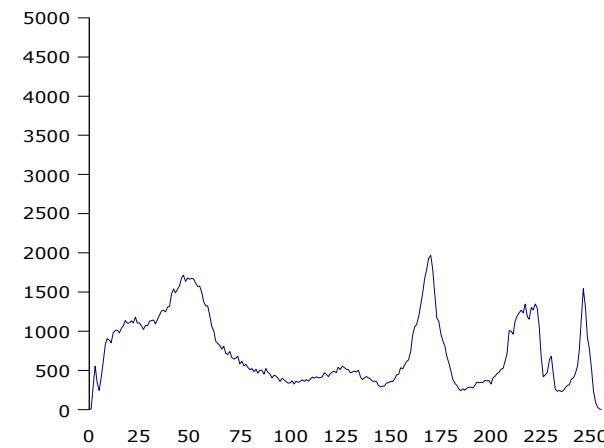


Decreasing the contrast

```
//Decrease contrast -50
for i=0:255
    LUT(i+1)=int(127+(i-127)*155/255+0.5);
    LUT(i+1)=min(255,max(0,LUT(i+1)));
end
image2=LUTgray(image,LUT);
imshow(image2);
```



skala Y obcięta





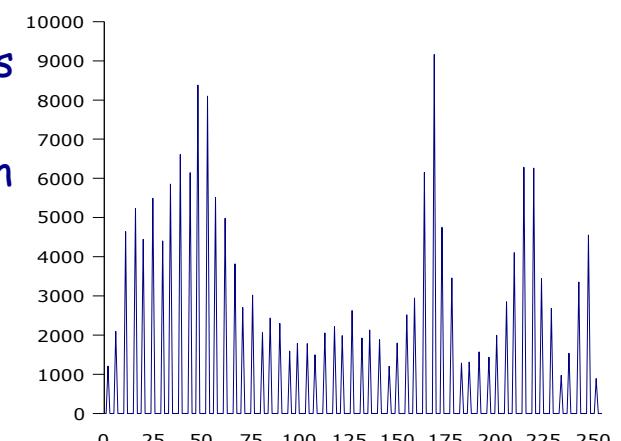
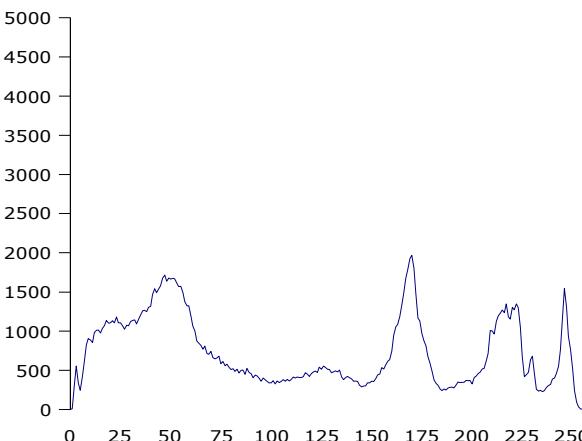
Loss of information when changing contrast

contrast -100

contrast +100



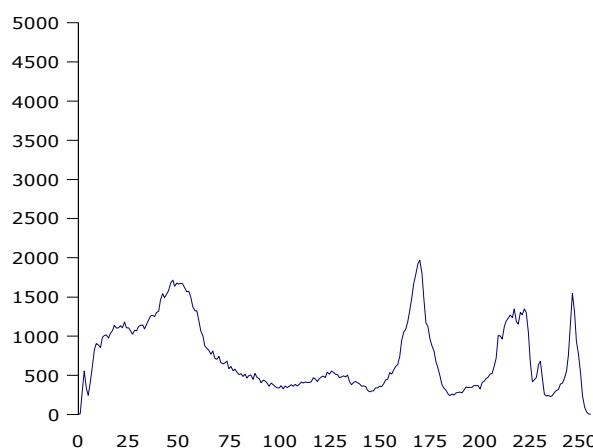
- seemingly the output image resembles the input
- but looking at the histograms, you can see that the **color depth** has been reduced



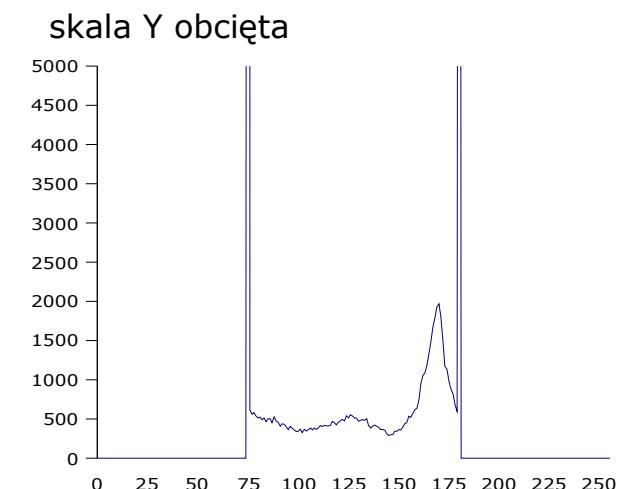


contrast +100

contrast -100



- the output image has "cropped" light and dark levels
- central levels remained unchanged





Gamma correction

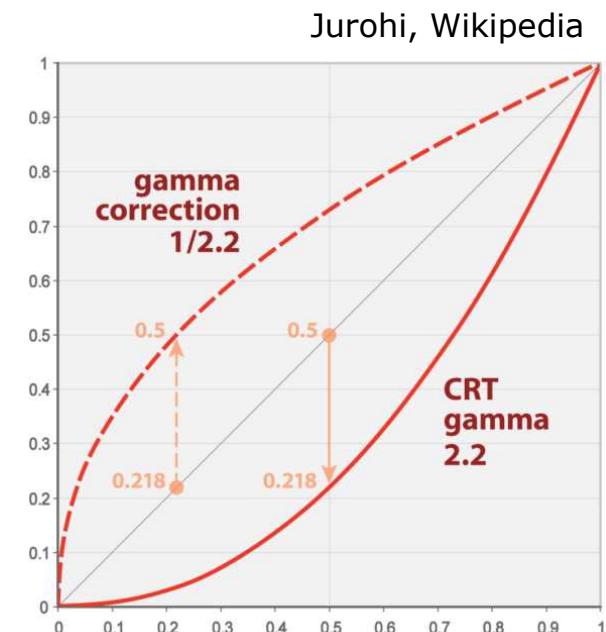
- change of intensity according to non-linear function

$$Image(x, y) \Rightarrow 255 \cdot \left(\frac{Image(x, y)}{255} \right)^{\gamma}$$

- a context-free operation, carried out as a LUT

```
for i=0:255
    LUT(i+1)=int(255*(i/255)^gamma+0.5);
end
image2=LUTgray(image,LUT);
imshow(image2);
```

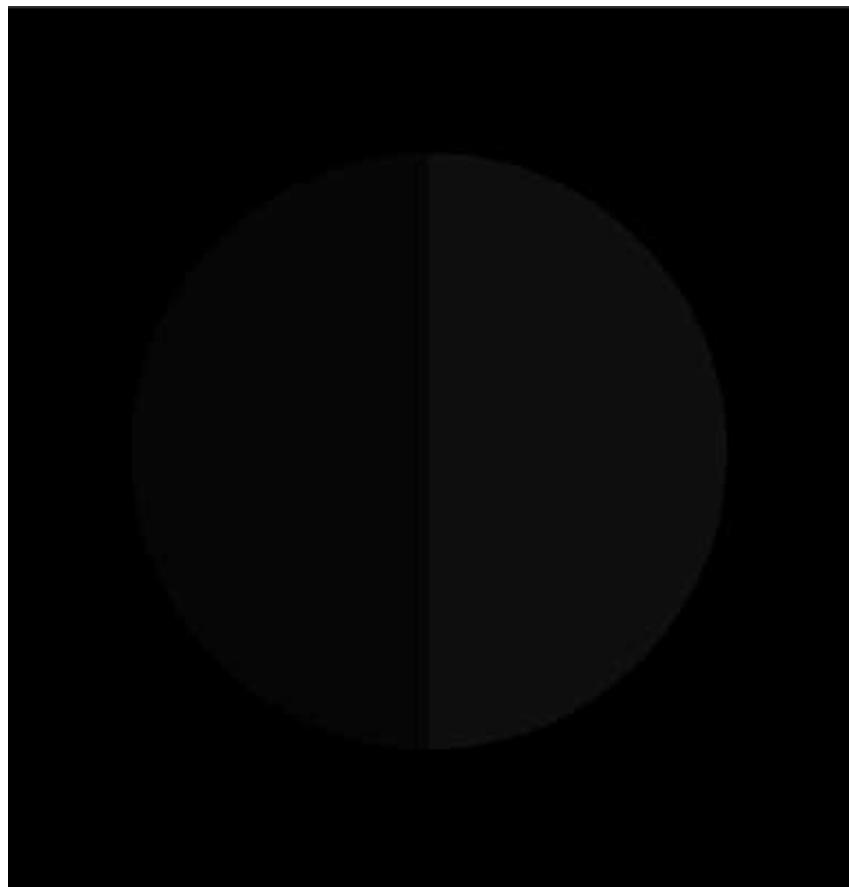
- very frequent adjust operation colors to non-linear characteristics CRT monitors
 - CRT monitors around 2.2
 - gamma correction 1 / 2.2



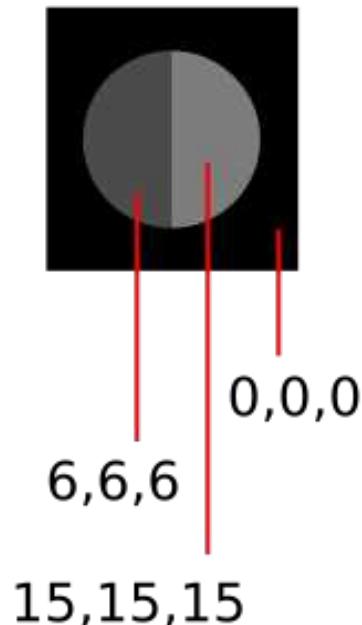


Test ustawienia korekcji gamma monitora

Janke, Wikipedia



RGB values apply to
large image; thumbnail
is exaggerated for clarity

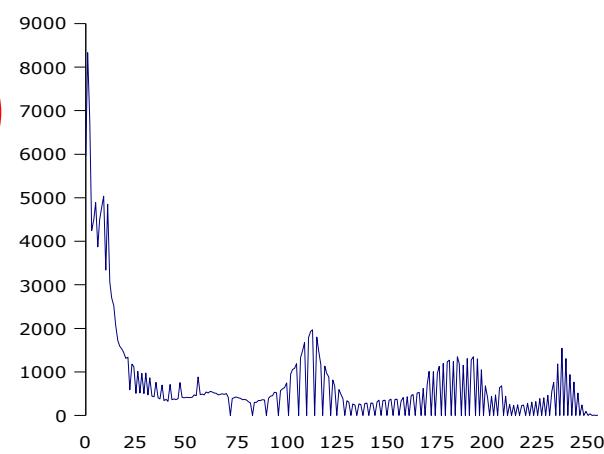
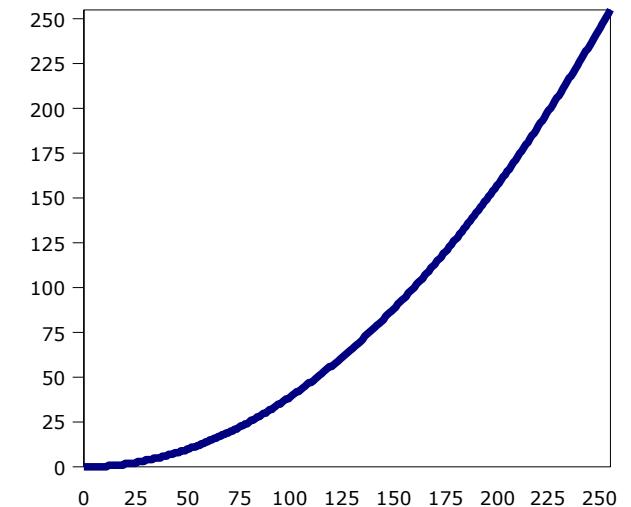
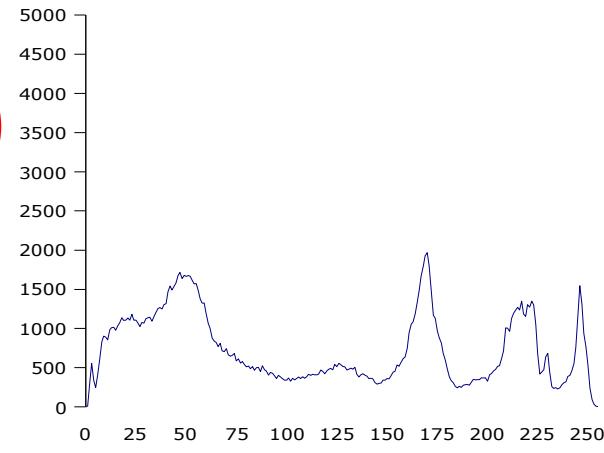


- lewe półkole powinno być praktycznie niewidoczne
- prawe półkole powinno być jak najbardziej widoczne



$\gamma > 1$

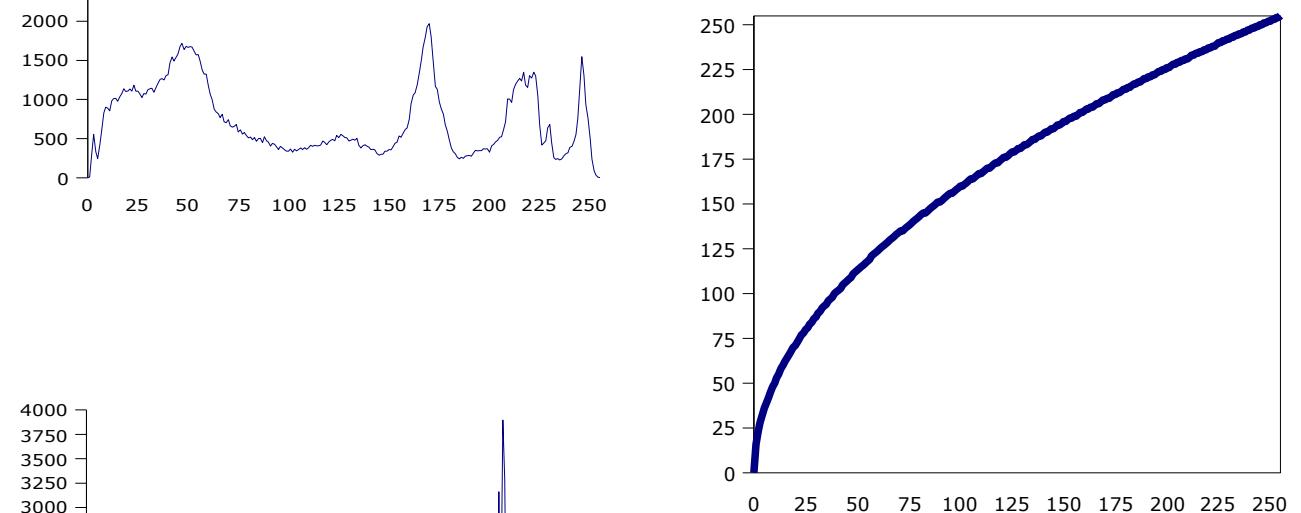
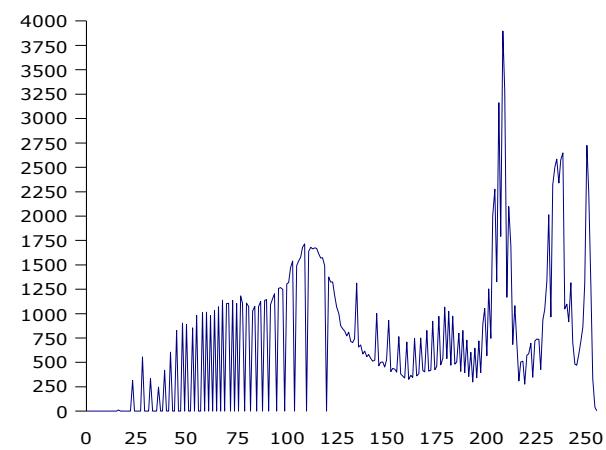
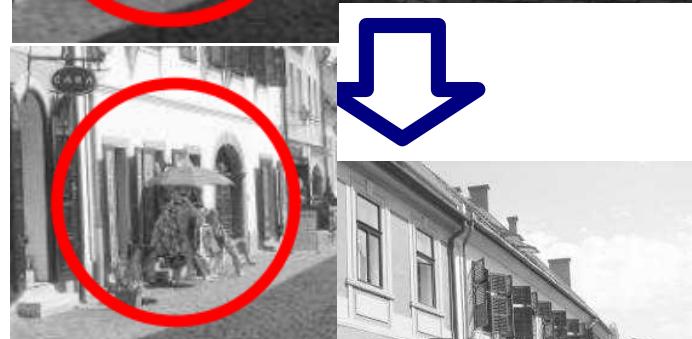
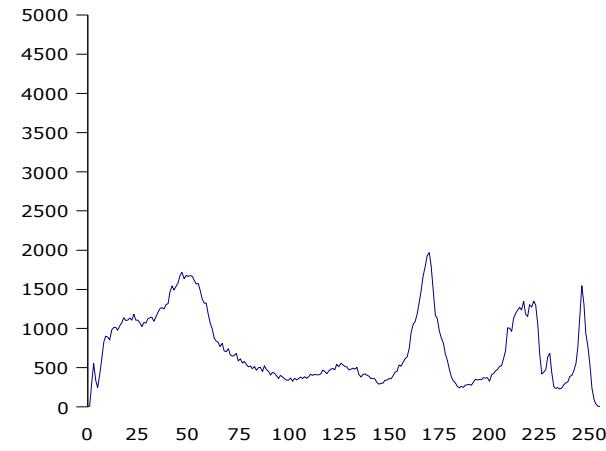
$\gamma = 2$ – gamma expansion, increasing the contrast of bright areas





$\gamma < 1$

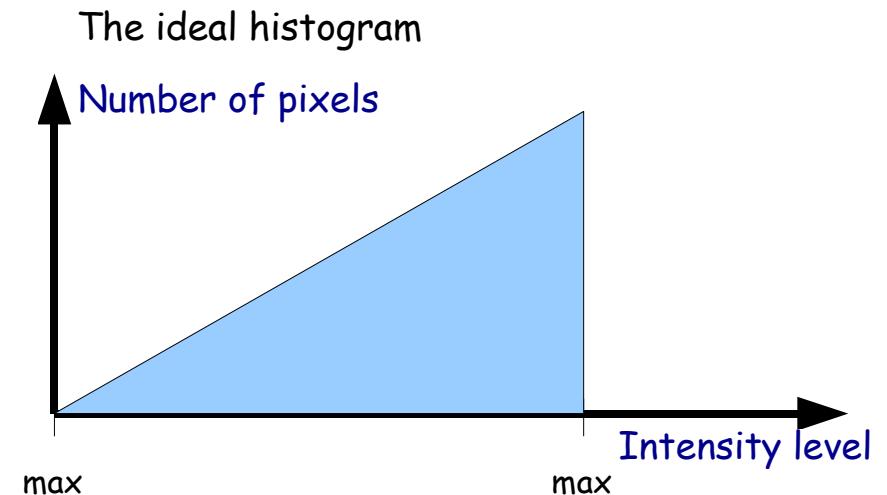
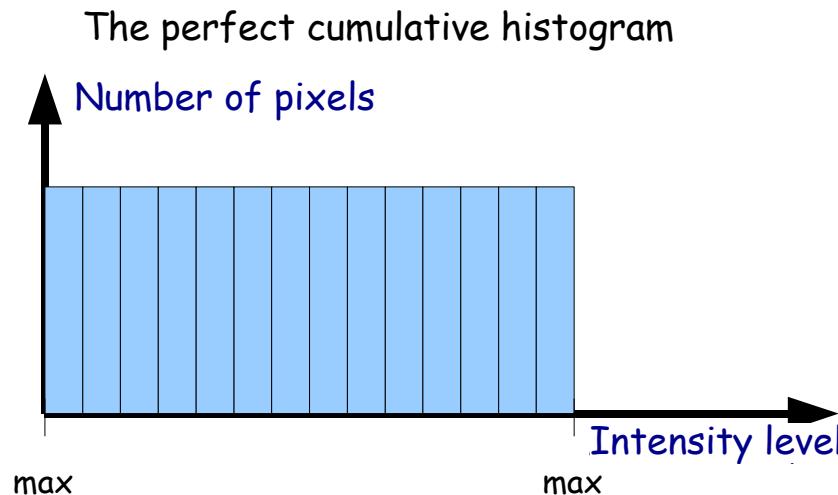
$\gamma = 0.5$ – gamma compression, increasing the contrast of dark areas





Histogram correction operation

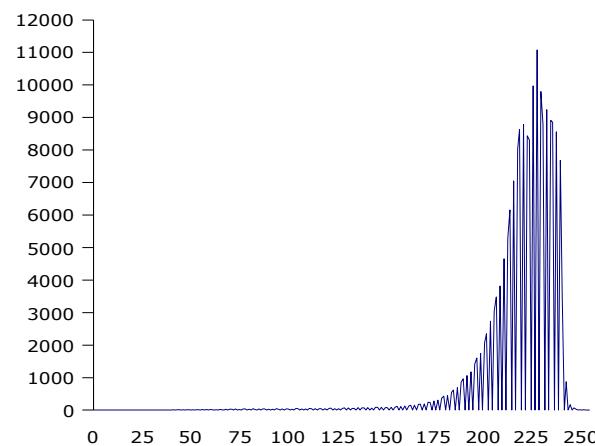
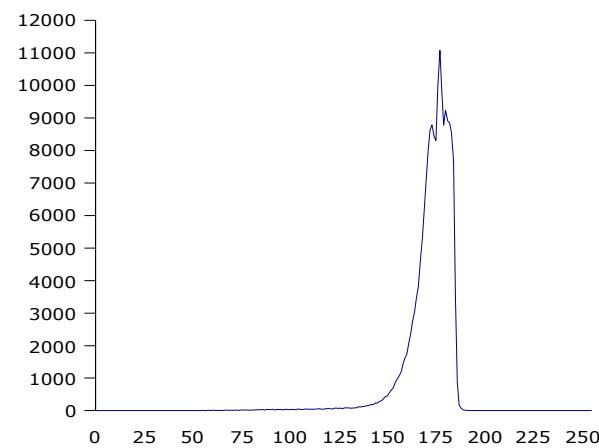
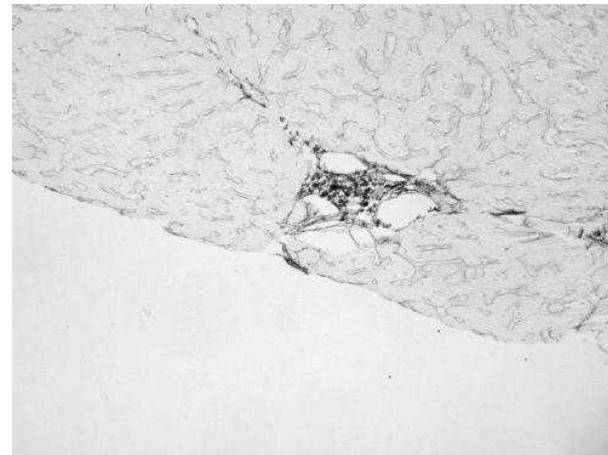
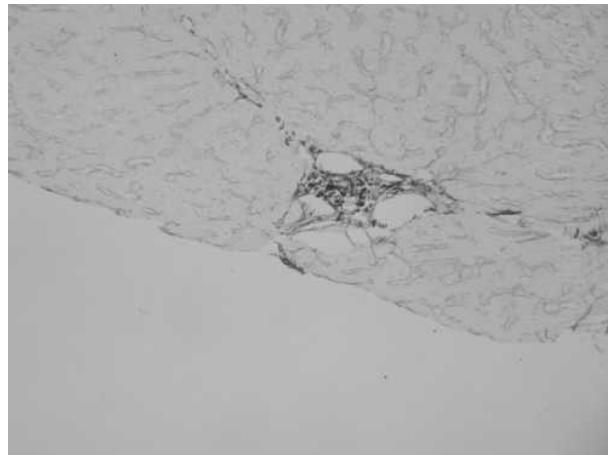
- are designed to improve the image contrast quality
- change the histogram
- extension of the histogram
 - pixels should use all available intensity levels
- equalization of the histogram
 - all levels should be approximately parallel
 - that is, the histogram should have been flat, without mountains and valleys





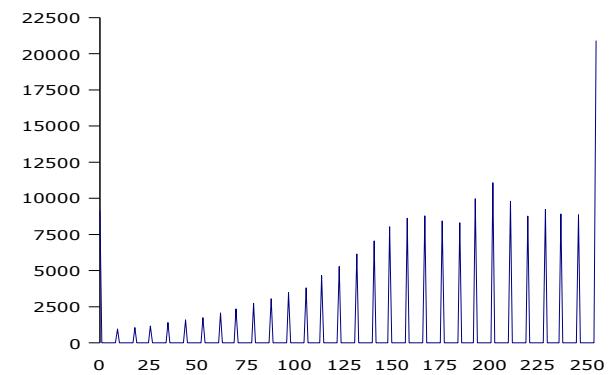
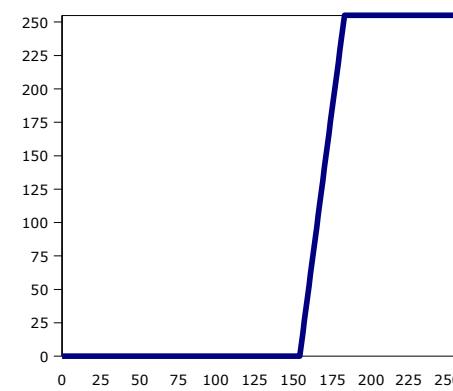
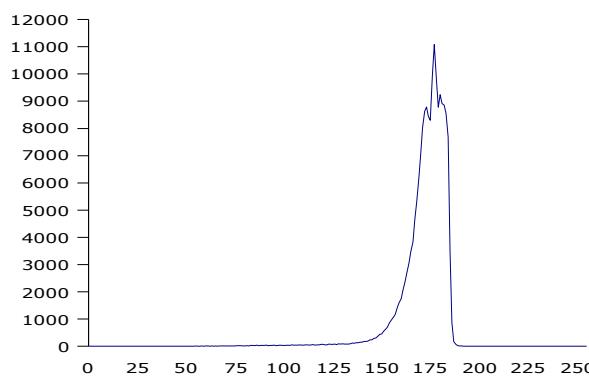
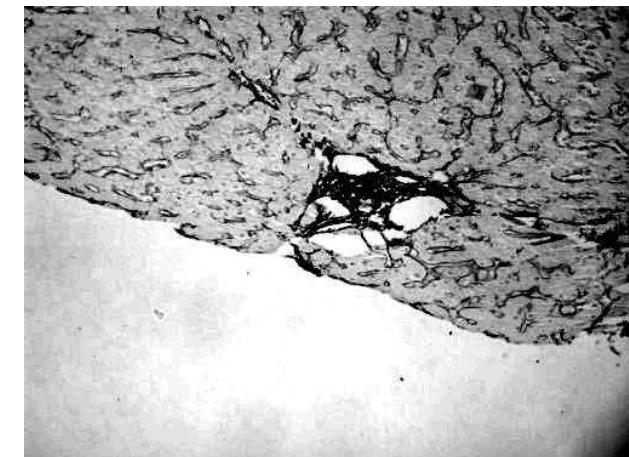
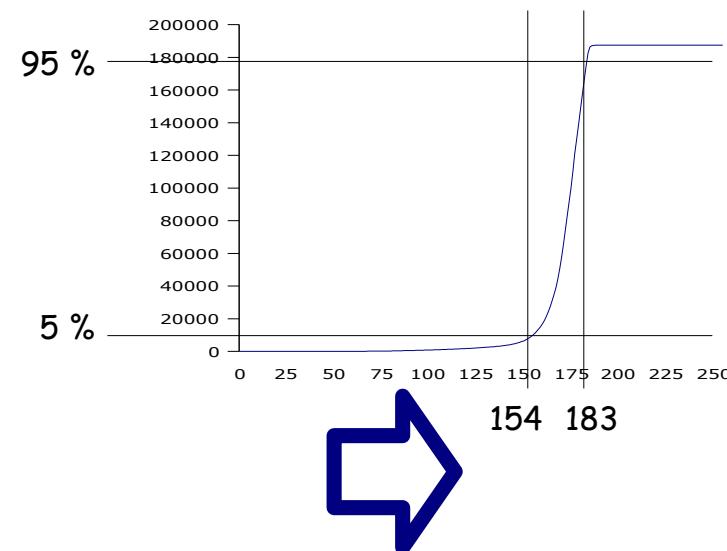
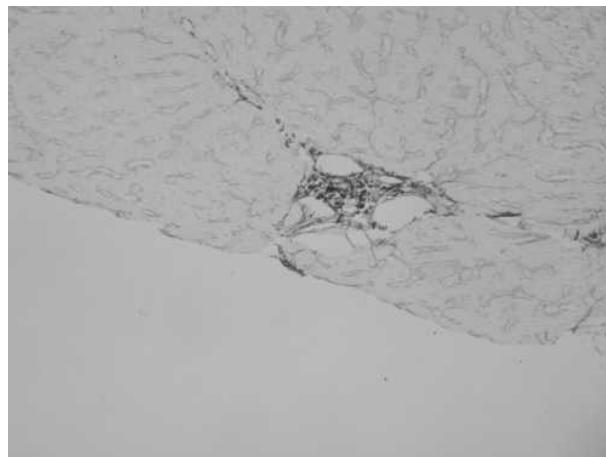
Stretching the histogram

$$I'(x, y) \Rightarrow (I(x, y) - \min) \cdot \frac{255}{\max - \min}$$



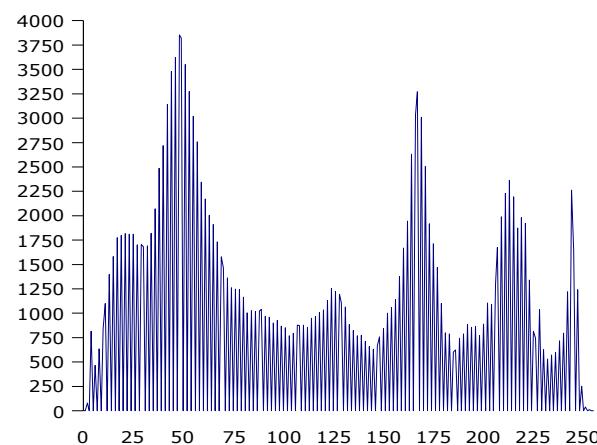
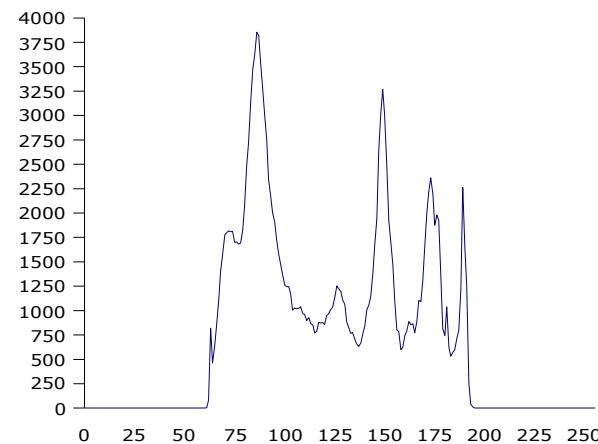


because the "tails" of the histogram can be very low and long - instead of max and min, for example, the quantiles 5% and 95% (percentiles) which can be calculated from the cumulative histogram





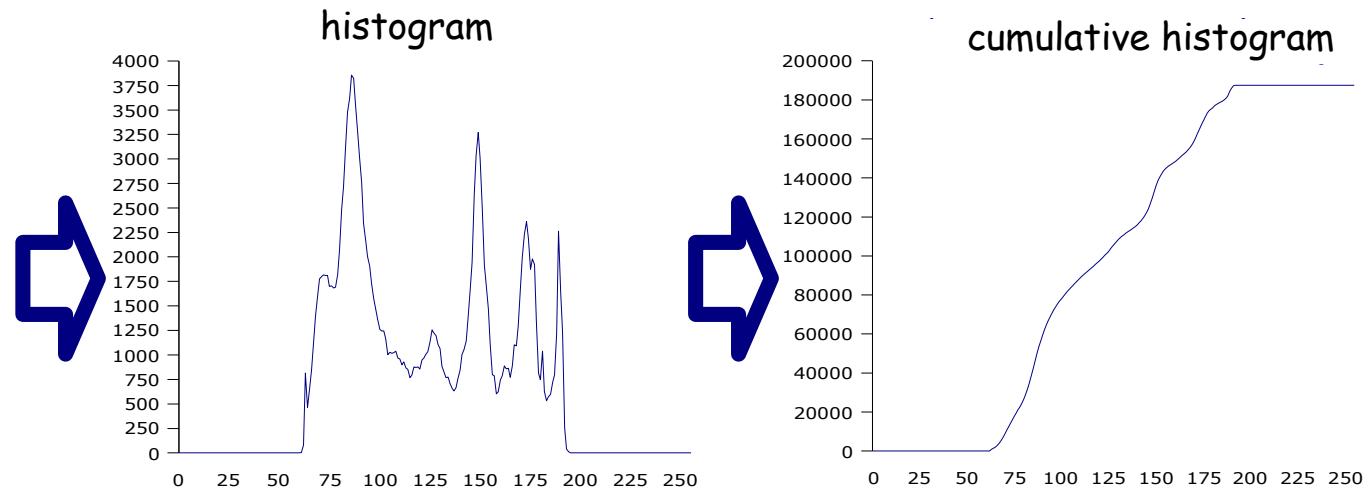
after stretching, they can still leave hills and valleys



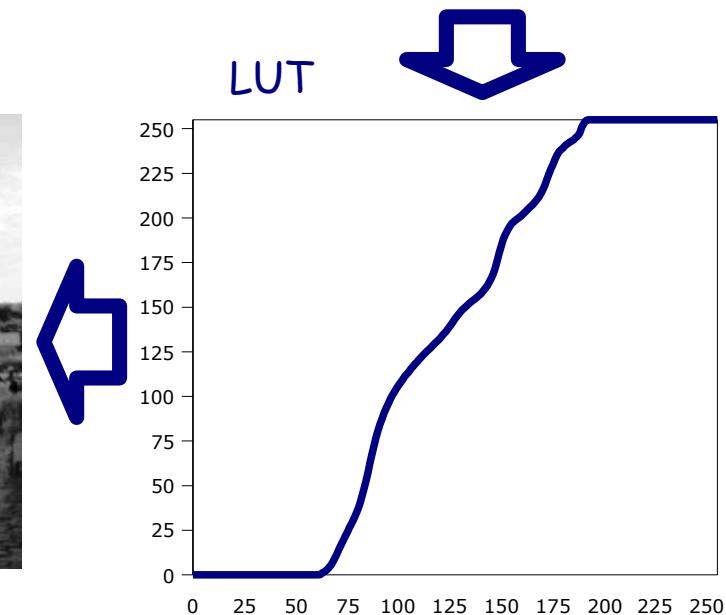
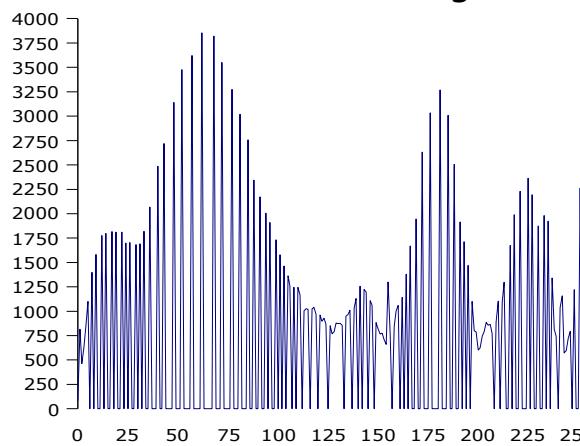


Alignment of the histogram

the cumulative histogram is taken as the LUT table



result histogram



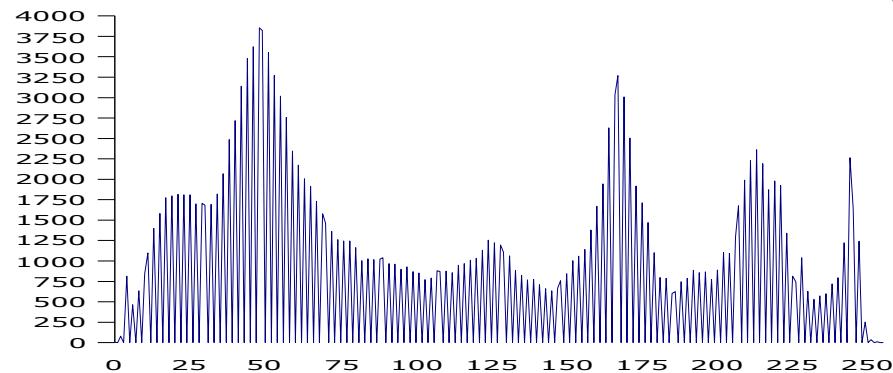
Stretch histogram versus histogram alignment



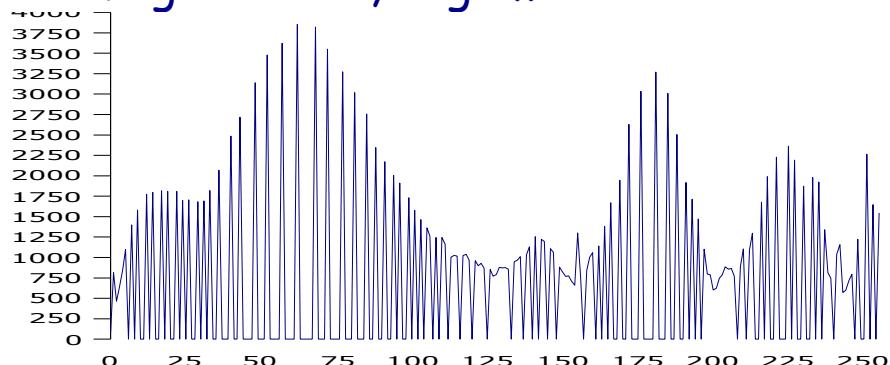
stretch histogram



histogram alignment



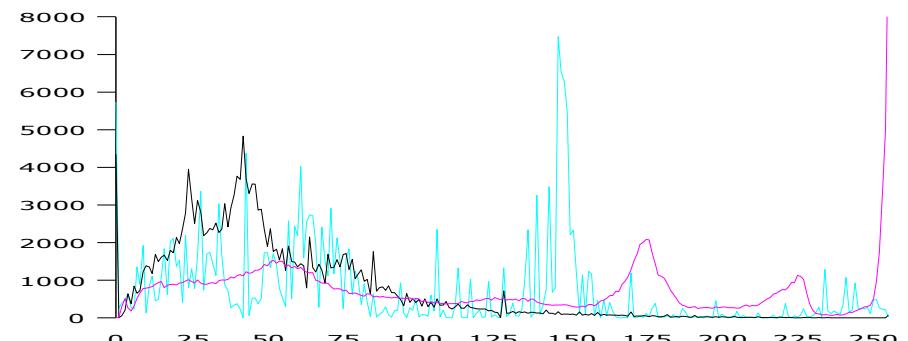
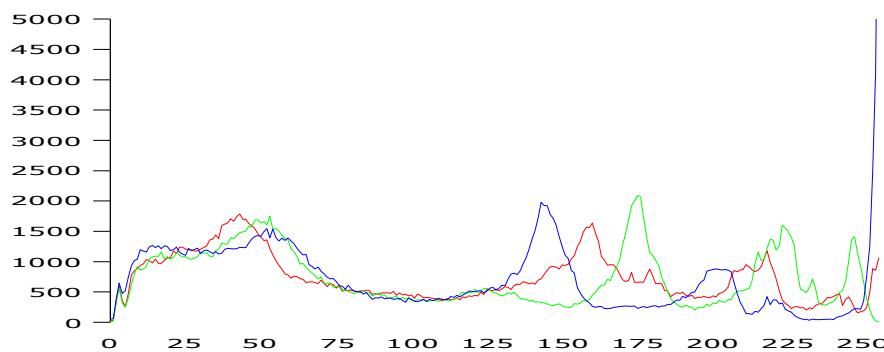
- alignment gives better results (such "smart" stretching), you can see more details
- aligned histogram is not ideal but better than in stretching
 - alignment:
 - extends "peaks"
 - compresses "valleys"
 - stretching is linear, alignment - non-linear





Operacja na histogramach obrazów kolorowych

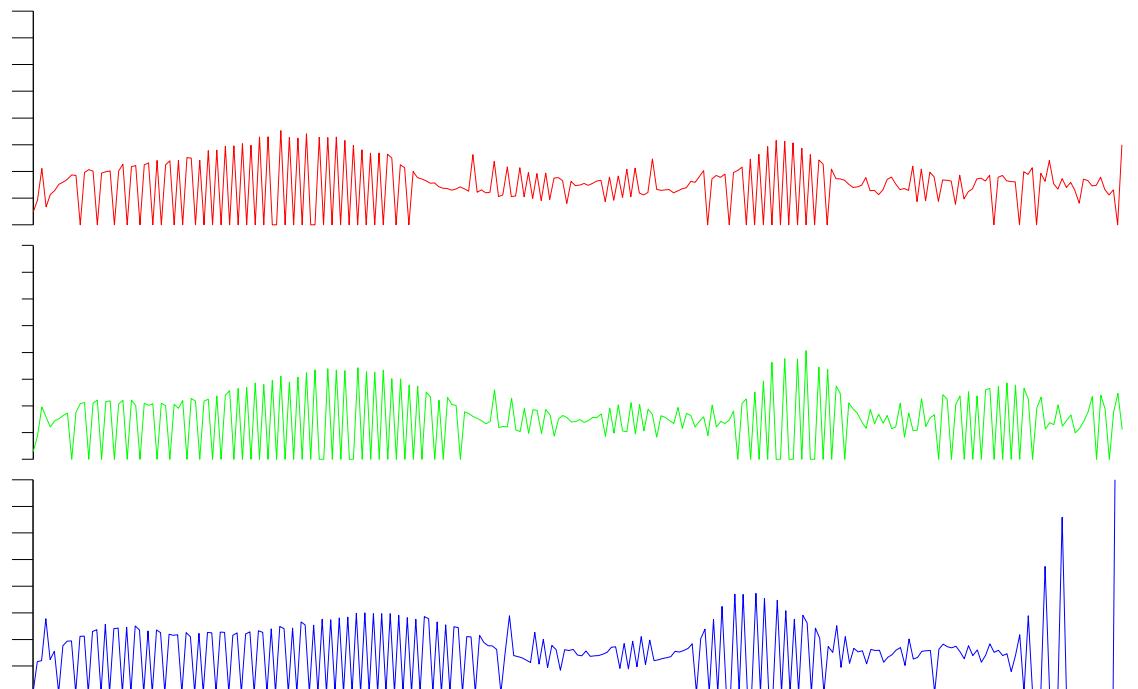
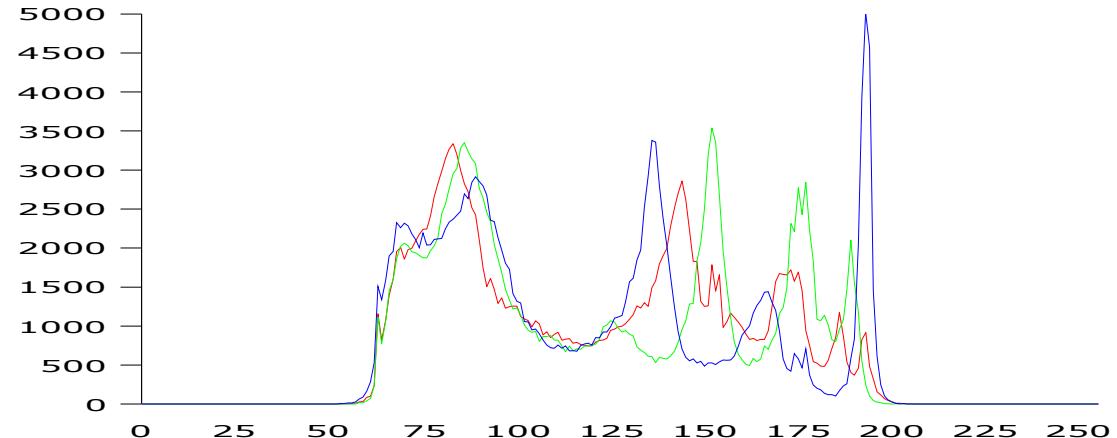
wykonywane na wszystkich kanałach (RGB, HSV) niezależnie lub z tymi samymi parametrami



Wyrównanie histogramu obrazu kolorowego



każdy kanał RGB wyrównywany niezależnie





Negatyw obrazu barwnego

- każdy kanał z RGB odwracany jest niezależnie

$$Image(x,y,kanał) = 255 - Image(x,y,kanał)$$

- operacja bezstratna – odwracalna





Zwiększenie intensywności kanału R

- dodanie do każdego piksela wartości (50,0,0)
- „zaczerwienienie” obrazu





Zmniejszenie intensywności kanału R

- odjęcie od każdego piksela wartości (50,0,0)
- dodanie składowej cyjan do obrazu





Zwiększenie intensywności kanału G

- dodanie do każdego piksela wartości (0,50,0)
- „zazielenienie” obrazu





Zmniejszenie intensywności kanału G

- odjęcie od każdego piksela wartości (0,50,0)
- dodanie składowej magenta (purpura, róż) do obrazu





Zwiększenie intensywności kanału B

- dodanie do każdego piksela wartości (0,0,50)
- „zaniebesczenie” obrazu





Zmniejszenie intensywności kanału B

- odjęcie od każdego piksela wartości (0,0,50)
- „zażółcenie” obrazu





Balans kolorów

- balans kolorów (skala każdego z kanałów RGB) decyduje o naturalności kolorów (zgodnością z percepcją człowieka w świetle naturalnym)
- różne oświetlenie zmienia balans kolorów, konieczna jest korekcja balansu
- w prostych urządzeniach odbywa się to poprzez „korekcję balansu bieli”
 - mierzy się kolor wzorcowej bieli w danym oświetleniu (R_0, G_0, B_0)
 - wszystkie kolory modyfikuje się wg równania macierzowego

$$\begin{bmatrix} R \\ G \\ B \end{bmatrix} \Rightarrow \begin{bmatrix} 255/R_0 & 0 & 0 \\ 0 & 255/G_0 & 0 \\ 0 & 0 & 255/B_0 \end{bmatrix} \begin{bmatrix} R \\ G \\ B \end{bmatrix}$$

- w bardziej zaawansowanych urządzeniach wykonuje się kalibrację kolorów wg tablicy kolorów wzorcowych – zależności są bardziej złożone niż powyżej



Zwiększenie jasności

- dodanie do kanału *Value* w modelu kolorów HSV stałej wartości
- barwy i nasycenie nie ulegają zmianie





Zmniejszenie jasności

- odjęcie od kanału *Value* w modelu kolorów HSV stałej wartości
- barwy i nasycenie nie ulegają zmianie





Zwiększenie nasycenie kolorów

- dodanie do kanału *Saturation* w modelu kolorów HSV stałej wartości
- barwy i jasność nie ulegają zmianie





Zmniejszenie nasycenie kolorów

- odjęcie od kanału *Saturation* w modelu kolorów HSV stałej wartości
- barwy i jasność nie ulegają zmianie



- ustawienie kanału *Saturation* na 0 tworzy obraz czarno-biały



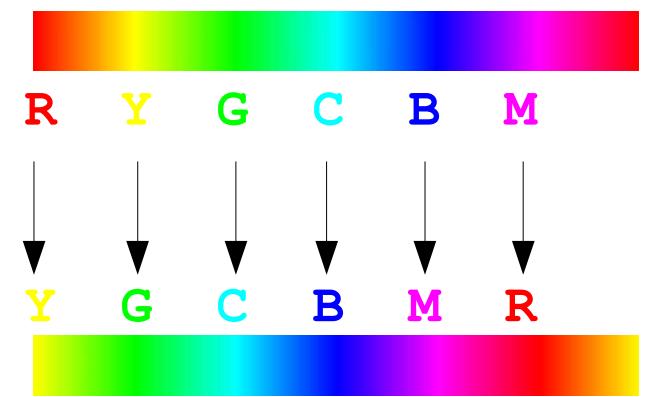
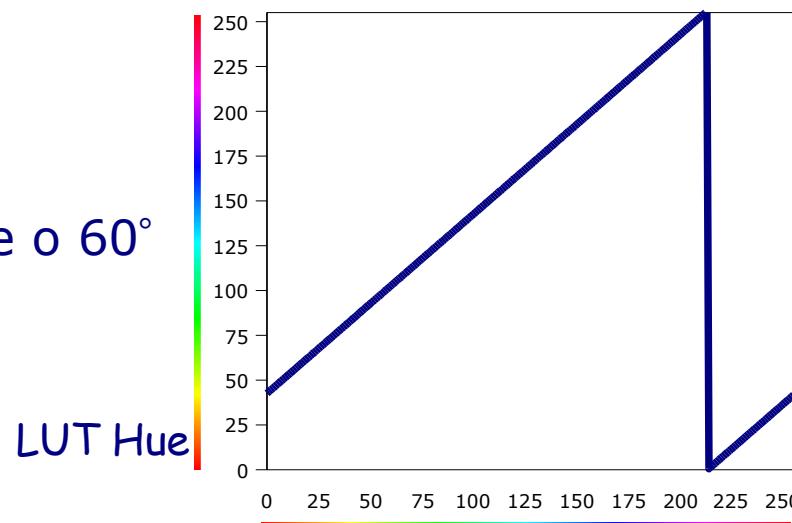


Przesunięcie kolorów

- cykliczna przesunięcie kanału barwy *Hue* w modelu kolorów HSV



- przesunięcie o 60°



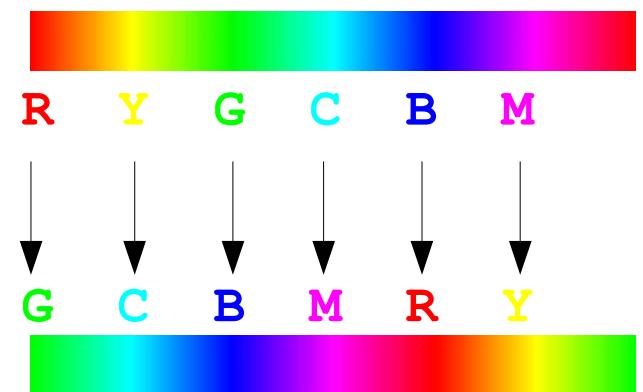
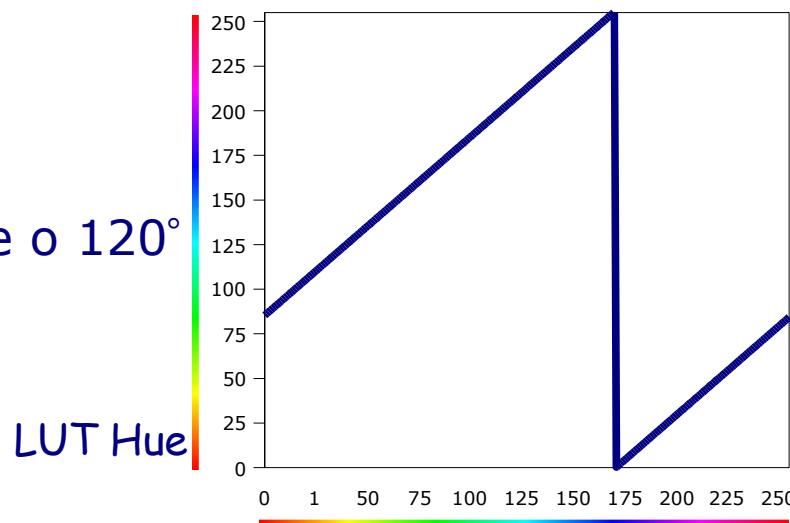


Przesunięcie kolorów

- cykliczna przesunięcie kanału barwy *Hue* w modelu kolorów HSV



- przesunięcie o 120°



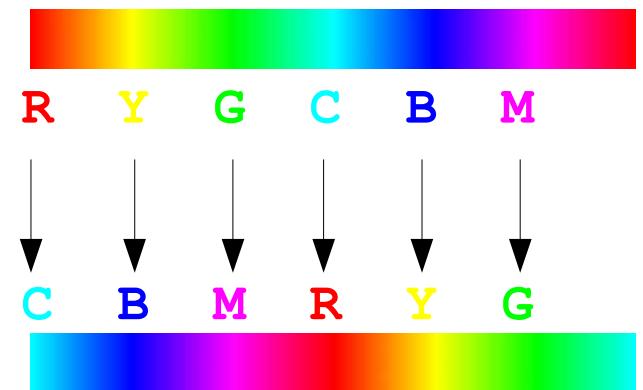
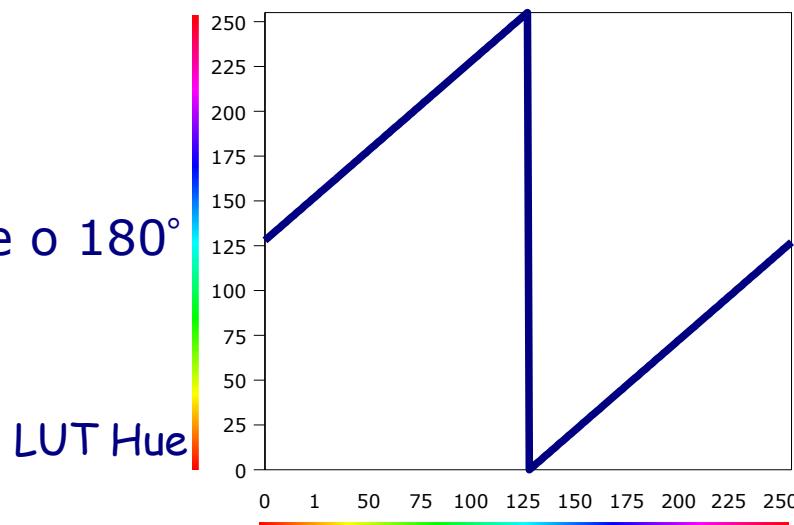


Przesunięcie kolorów

- cykliczna przesunięcie kanału barwy *Hue* w modelu kolorów HSV



- przesunięcie o 180°



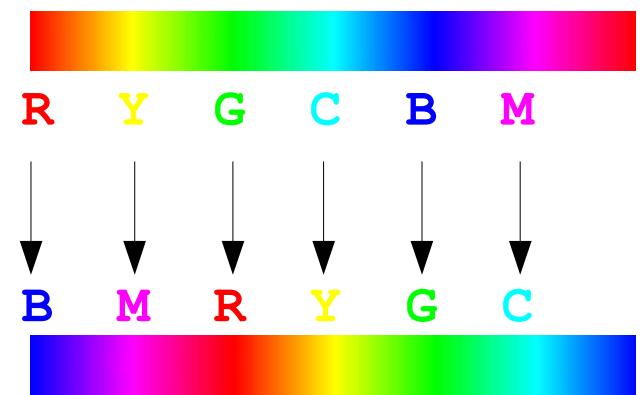
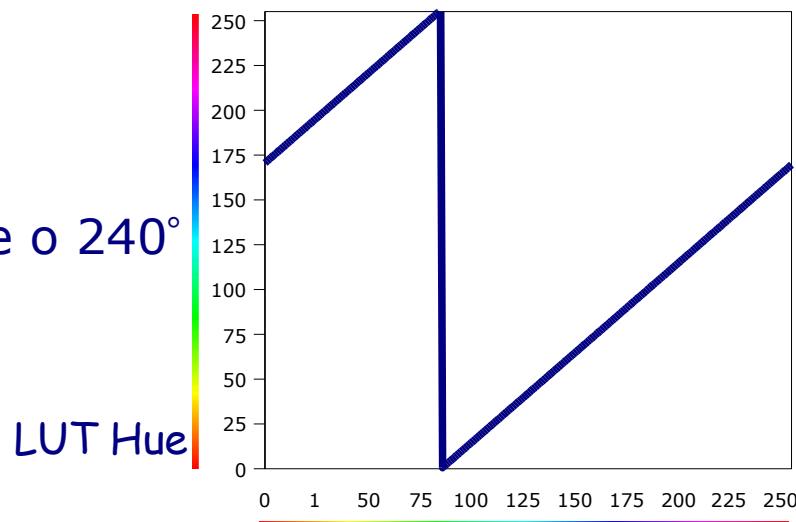


Przesunięcie kolorów

- cykliczna przesunięcie kanału barwy *Hue* w modelu kolorów HSV



- przesunięcie o 240°



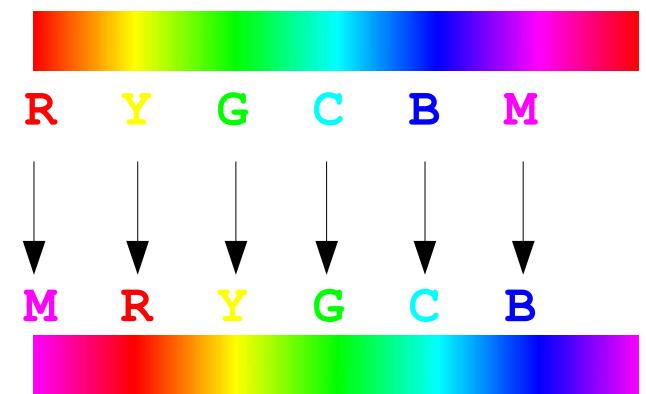
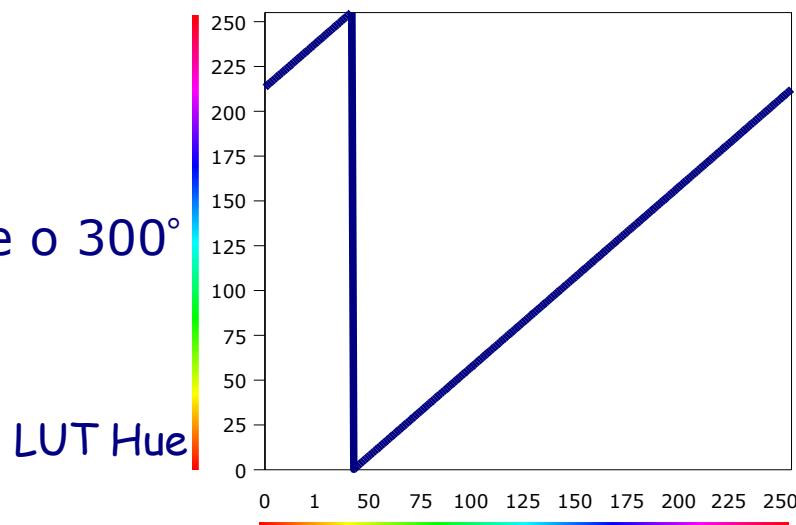


Przesunięcie kolorów

- cykliczna przesunięcie kanału barwy *Hue* w modelu kolorów HSV



- przesunięcie o 300°





Arytmetyka obrazów

- z dwóch lub więcej obrazów powstaje jeden wynikowy
- zwykle stosowana do obrazów w skali szarości lub binarnych (ale nie tylko)

$Image1(x,y) \times Image2(x,y) \Rightarrow Image1(x,y) \text{ operation } Image2(x,y)$

- dodawanie obrazów
- odejmowanie obrazów
- mnożenie obrazów
- dzielenie obrazów
- operacje bitowe (OR, AND, XOR, NOT)



Dodawanie obrazów

- efekt podwójnej ekspozycji

$$Image(x,y) = Image1(x,y) + Image2(x,y)$$

lub

$$Image(x,y,kanał) = Image1(x,y,kanał) + Image2(x,y,kanał)$$

- może pojawić się problem z wartościami powyżej maksymalnej wartości
- różne sposoby poradzenia sobie z tym zjawiskiem:
 - przeskalowanie obrazu wynikowego do oryginalnej skali
$$Image(x,y) = (Image1(x,y) + Image2(x,y)) / 2$$
 - ustawienie wartości przekraczających *maks* na *maks*
$$Image(x,y) = \min(maks, Image1(x,y) + Image2(x,y))$$
 - efekt cykliczności – wartość wynikowa modulo *maks*
$$Image(x,y) = (Image1(x,y) + Image2(x,y)) \% 256$$
- zastosowania:
 - efekt artystyczny
 - nakładanie tekstur
 - usuwanie szumu poprzez uśrednianie (dodanie proporcjonalne) kolejnych obserwacji tej samej sceny
 - uwypuklenie pewnych cech po dodaniu ich obrazu (np. wyostrzanie przez dodanie krawędzi)
 - dodanie stałej wartości rozjaśnia obraz



Proporcjonalne dodawanie obrazów

$$Image(x,y) = (Image1(x,y) + Image2(x,y)) / 2$$



+



||





Ważone dodawanie obrazów

$$Image(x,y) = W * Image1(x,y) + (1-W) * Image2(x,y), \quad 0 \leq W \leq 1$$

W=0,3



W=0,7



- automatyczne wyskalowanie obrazu wynikowego
- efekt przejścia między 2 obrazami płynnie zmieniając W: $0 \rightarrow 1$



Kanał alfa obrazów

- czwarty kanał, obok RGB, HSV, ...
(lub drugi dla obrazów monochromatycznych)
- określa, w którym miejscu obraz jest przezroczysty (transparentny, najczęściej reprezentowany przez kolor czarny, waga $W=0$) a w którym nieprzezroczysty (ang. *solid, opaque*, reprezentowany zwykle przez kolor biały, waga $W=1$)
- obraz przezroczysty „przepuszcza” część obrazu pod nim w stopniu określonym przez alfę (przezroczystość)
- ma to znaczenie przy nakładaniu się kilku obrazów



Kanał alfa obrazów

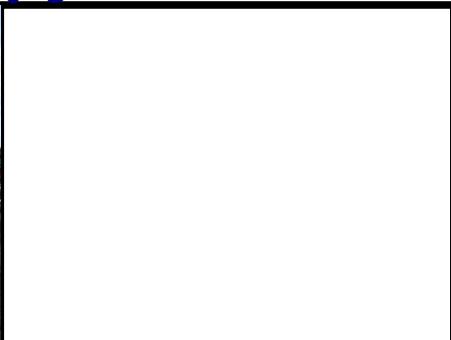
obraz + jego kanał alfa



daje następujący wynik



nałożony na ten obraz+jego kanał alfa





Dodawanie obrazów z saturacją

$$Image(x,y) = \min (255, Image1(x,y) + (1-W) * Image2(x,y))$$



- mało czytelne obrazy
- regiony dominujące
- dużo obszarów z efektem saturacji - białych



Dodawanie obrazów z „zawinięciem” wartości

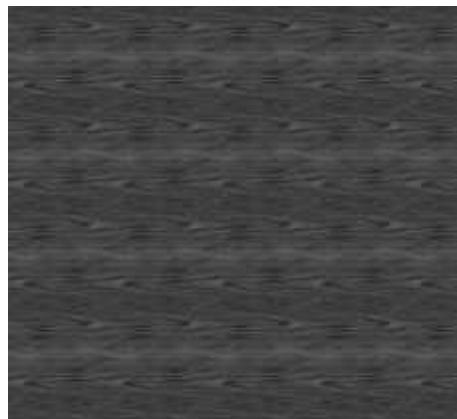
$$Image(x,y) = \text{modulo} (Image1(x,y) + (1-W) * Image2(x,y), 256)$$



- można odnaleźć więcej szczegółów z obu obrazów
- nadal mało naturalny obraz



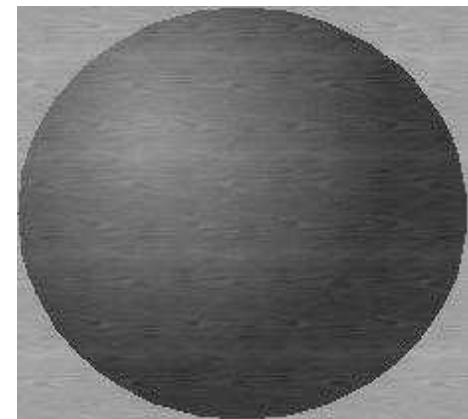
Dodanie krawędzi do obrazu



+



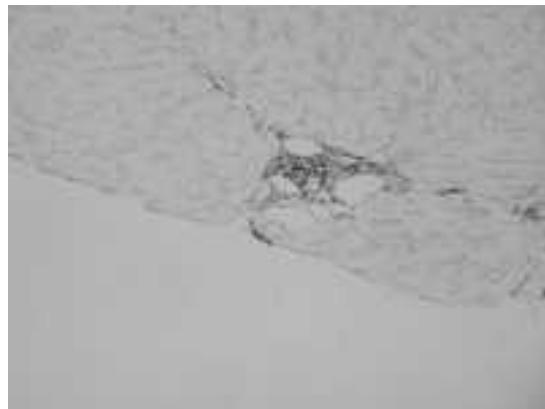
=



- dodawanie proporcjonalne
- obraz w dole po operacji dodania z saturacją tła oryginalnego obrazu kuli
(tło uzyskane przez progowanie)



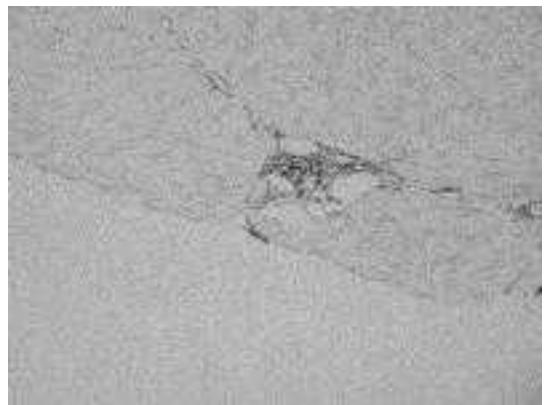
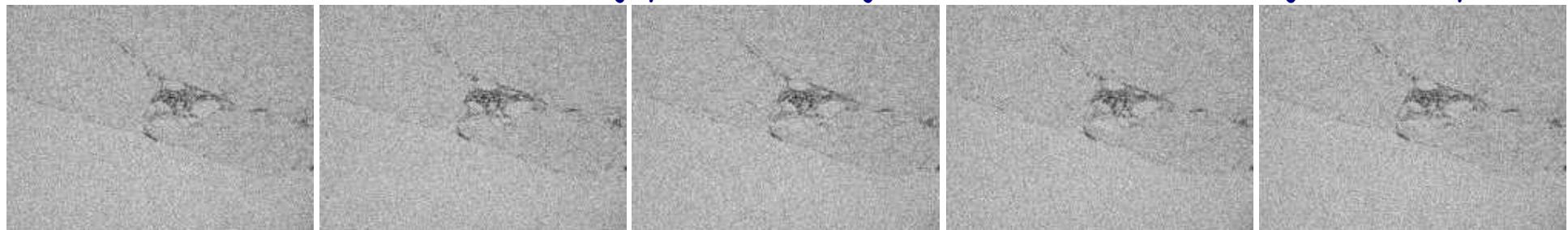
Usuwanie szumu przez uśrednianie



Obraz oryginalny

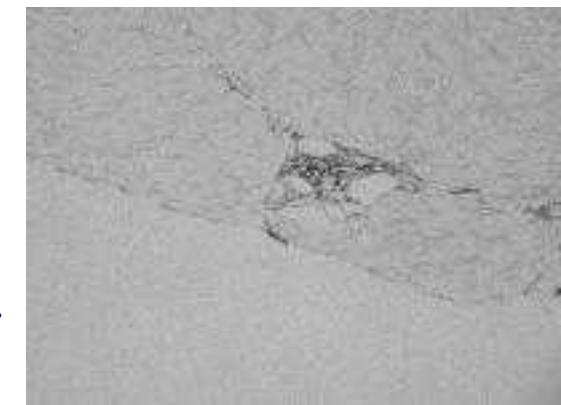
- nazywane uśrednianiem czasowym
- szum musi mieć średnią zero i stałą wariancję

5 kolejnych obserwacji z urządzenia które dodaje znaczący szum



Dodanie proporcjonalne
5 powyższych obserwacji
(szum zmalał)

Dodanie proporcjonalne
10 obserwacji





Wystrzanie obrazu przez dodanie krawędzi



Obraz oryginalny (rozmyty)

+

||



Obraz krawędzi (Laplasjan)
po wyrównaniu histogramu
dla lepszej wizualizacji



Obraz wynikowy po dodaniu
obrazu oryginalnego
i krawędzi
(tak naprawdę
po 2 takich operacjach)



Odejmowanie obrazów

$$Image(x,y) = Image1(x,y) - Image2(x,y)$$

lub

$$Image(x,y,kanał) = Image1(x,y,kanał) - Image2(x,y,kanał)$$

- może pojawić się problem z wartościami ujemnymi
- różne sposoby poradzenia sobie z tym zjawiskiem:
 - wartość bezwzględna różnicy
$$Image(x,y) = | Image1(x,y) - Image2(x,y) |$$
 - ustawienie wartości ujemnych na 0
$$Image(x,y) = \max(0, Image1(x,y) - Image2(x,y))$$
 - efekt cykliczności – wartość wynikowa modulo maks
$$Image(x,y) = (Image1(x,y) - Image2(x,y) + 256) \% 256$$
- operacja w zasadzie asymetryczna
- zastosowania:
 - obrazy różnicowe – wyodrębnianie różnic między obrazami podobnych obiektów lub tego samego obiektu w różnych obrazach
 - odejmowanie od obrazu niejednorodnego tła – w sytuacji gdy np. oświetlenie przy akwizycji było nierówne
 - odjęcie stałej wartości zmniejsza jasność obrazu



Odejmowanie – desaturacja

$$Image(x,y) = \max(0, Image1(x,y) - Image2(x,y))$$



- dużo regionów czarnych - nieczytelnych



Odejmowanie – desaturacja

$$Image(x,y) = \max(0, Image2(x,y) - Image1(x,y))$$



- operacja komplementarna – inny obiekt jest pierwszoplanowy, a inny jest tłem



Odejmowanie cykliczne

$$Image(x,y) = \text{modulo} (Image1(x,y) - Image2(x,y), 256)$$



- więcej widocznych szczegółów
- efekt nieczytelny – co widać na tym obrazie ?



Odejmowanie cykliczne

$$Image(x,y) = \text{modulo} (Image2(x,y) - Image1(x,y), 256)$$



- operacja komplementarna – negatyw poprzedniej



Odejmowanie bezwzględne

$$Image(x,y) = | Image2(x,y) - Image1(x,y) |$$



- operacja symetryczna
- widać 2 obrazy, trochę jak w dodawaniu



Obrazy różnicowe

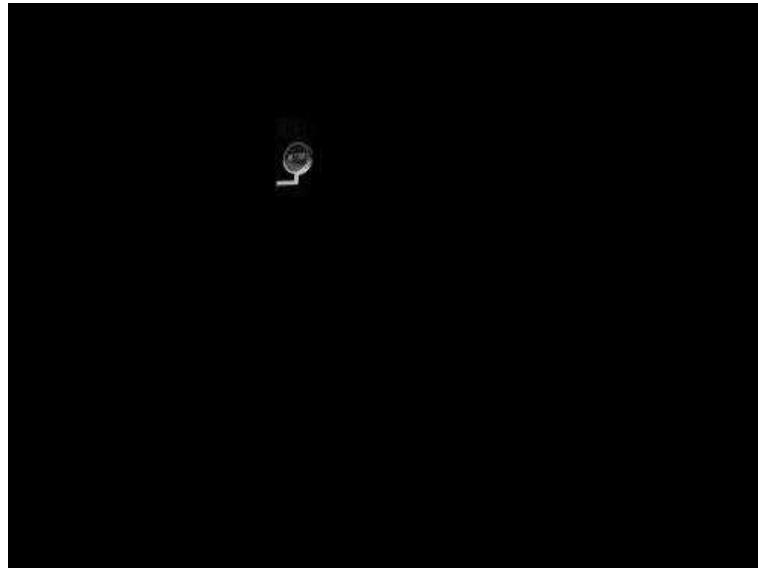


- zagadka – czym różnią się te dwa obrazy ?

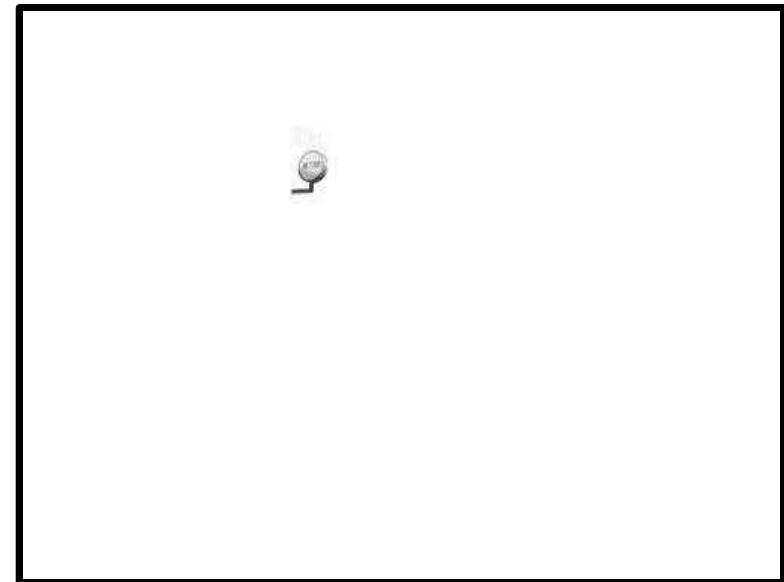


Obrazy różnicowe

Obraz różnicowy (różnica bezwzględna)



Negatyw różnicy

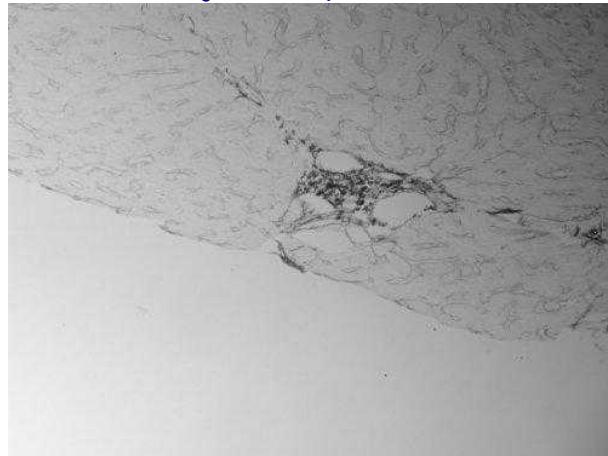


- obrazy różnicowe wykazują różnice
- często wykorzystywana technika w ocenie jakości segmentacji obrazów



Odejmowanie tła

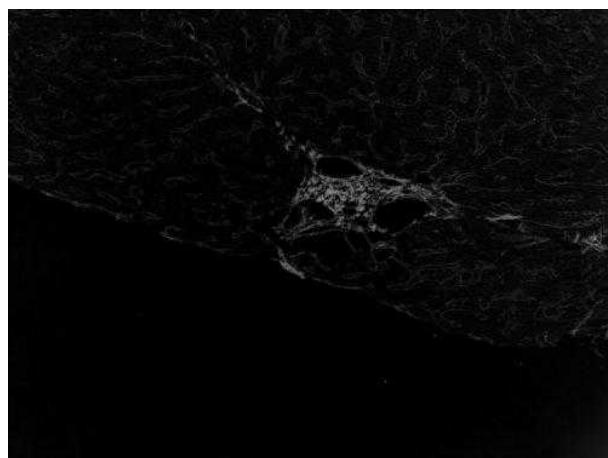
Obraz wejściowy



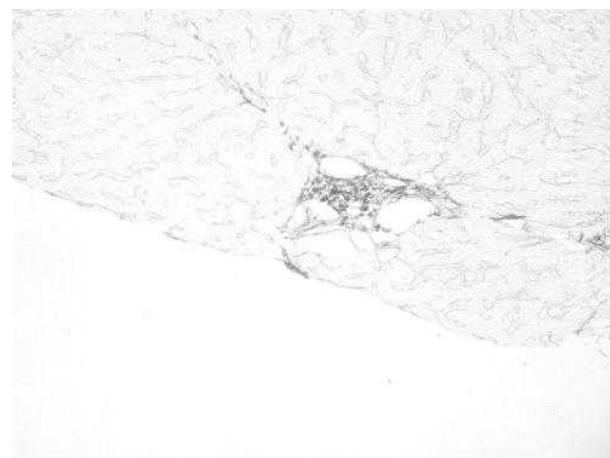
Kamera filmuje jednolite tło



Odjęcie tła od obrazu z obcięciem wartości



Negatyw odejmowania





Mnożenie obrazów

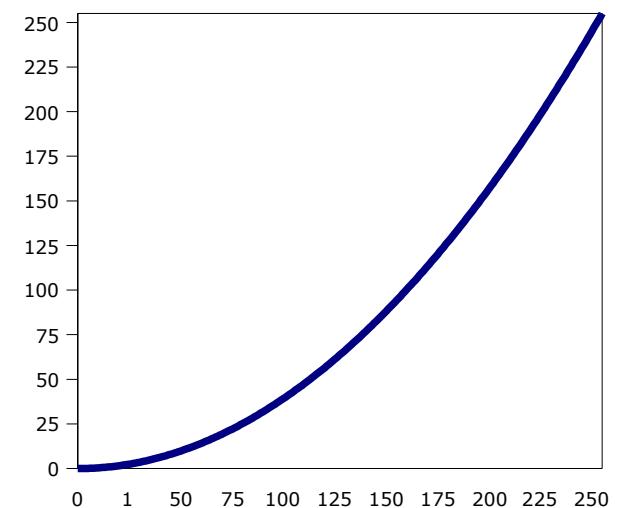
$$Image(x,y) = Image1(x,y) * Image2(x,y)$$

lub

$$Image(x,y,kanał) = Image1(x,y,kanał) * Image2(x,y,kanał)$$

- jak w dodawaniu może się pojawić problem z przekraczaniem zakresu
- rzadko stosuje się mnożenie 2 rzeczywistych, różnych obrazów
- wyjątek – mnożenie przez obraz binarny – nakładanie maski, ale lepiej nadają się tu bitowe operacje (AND)
- częściej mnożenie obrazu przez liczbę – rozjaśnia to obraz lepiej niż dodanie do obrazu stałej
- jeśli składowe obrazu reprezentowane są przez wartości z przedziału $[0,1]$ wynik mnożenia nie wykracza z zakresu
- mnożenie takiego obrazu przez samego siebie powoduje pociemnienie obrazu

LUT potęgowania obrazu
z wartościami z zakresu $[0,1]$
(wykres $y=x^2$ w przedziale $(0,1)$)





Potęgowanie obrazów

- potęgowanie obrazu którego wartości należą do zakresu $[0,1]$ – obraz ciemnieje, wartości nie wychodzą z zakresu



- potęgowanie obrazu którego wartości należą do zakresu $[0,255]$ – wartości prawie wszędzie wychodzą z zakresu (tu: obcięcie wartości poza zakresem – saturacja)

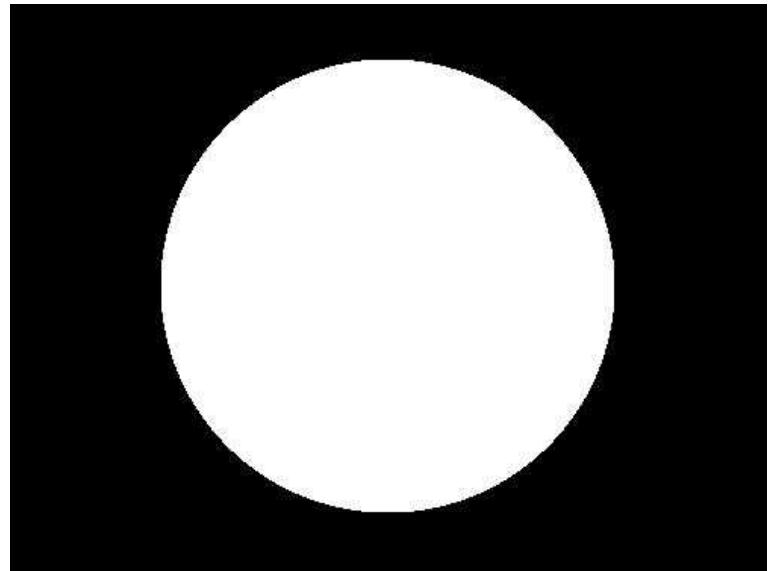




Maskowanie binarne obrazów



*



||



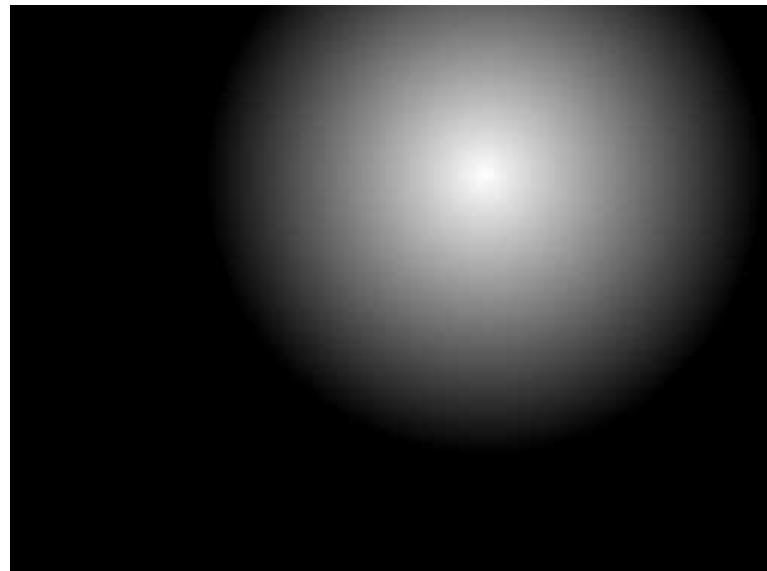
0 - czarny
1 - biały



Maskowanie ciągłe obrazów



*



||



0 - czarny
1 - biały



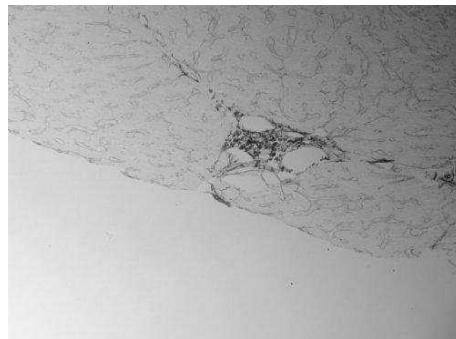
Dzielenie obrazów

$$Image(x,y) = Image1(x,y) / Image2(x,y)$$

lub

$$Image(x,y,kanał) = Image1(x,y,kanał) / Image2(x,y,kanał)$$

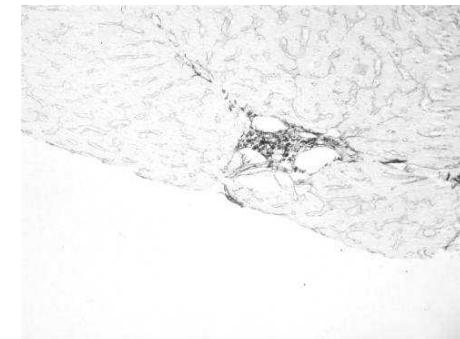
- uwaga na dzielenie przez zero !!!
- podobne zastosowanie do odejmowania obrazów (lepsze efekty)
 - dzielenie przez tło



/



=



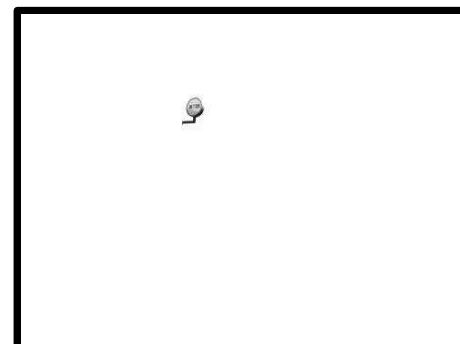
- wykrywanie w ruchu obrazu



/



=

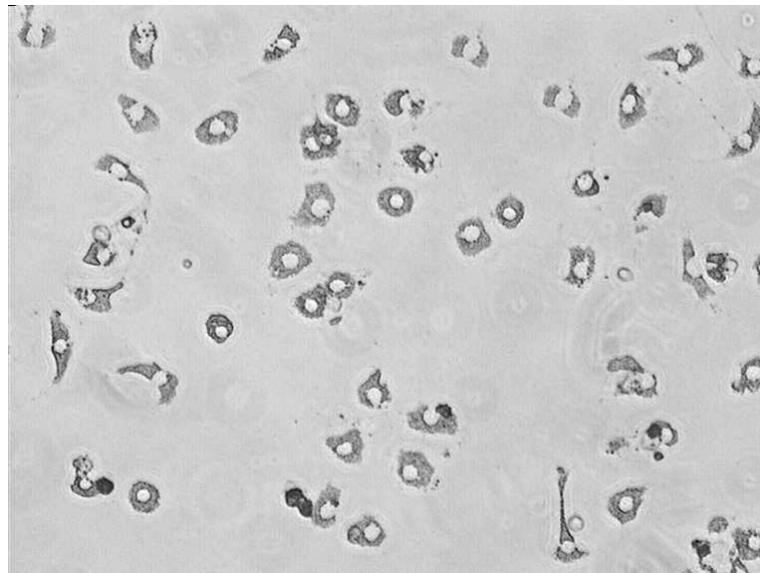




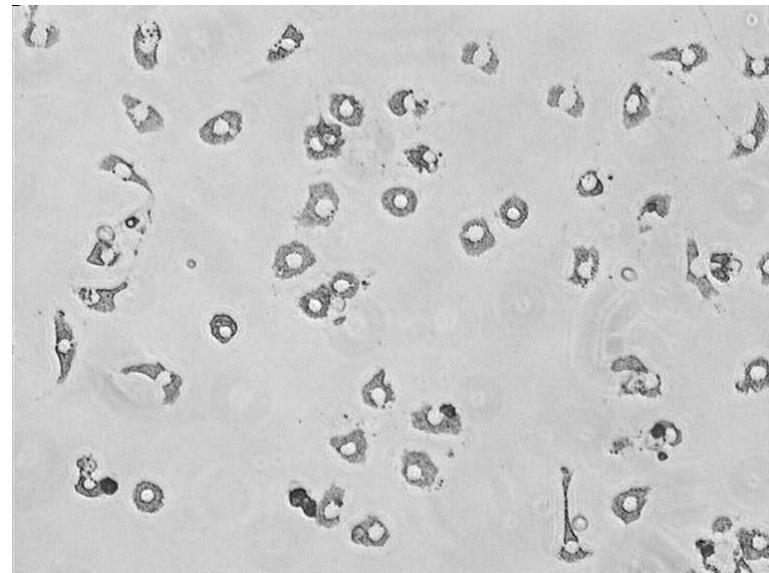
Dzielenie obrazów w wykrywaniu ruchu

jedna z komórek poruszyła się, która ?

Obraz I1



Obraz I2



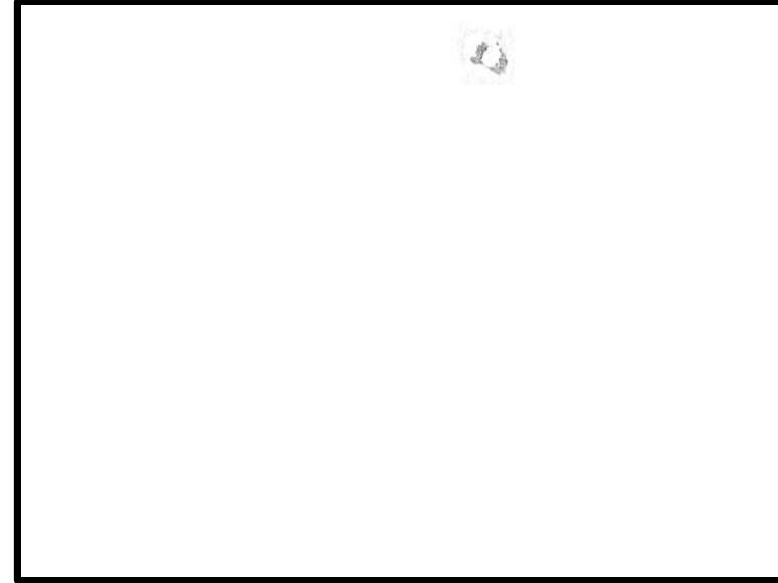


Dzielenie obrazów w wykrywaniu ruchu

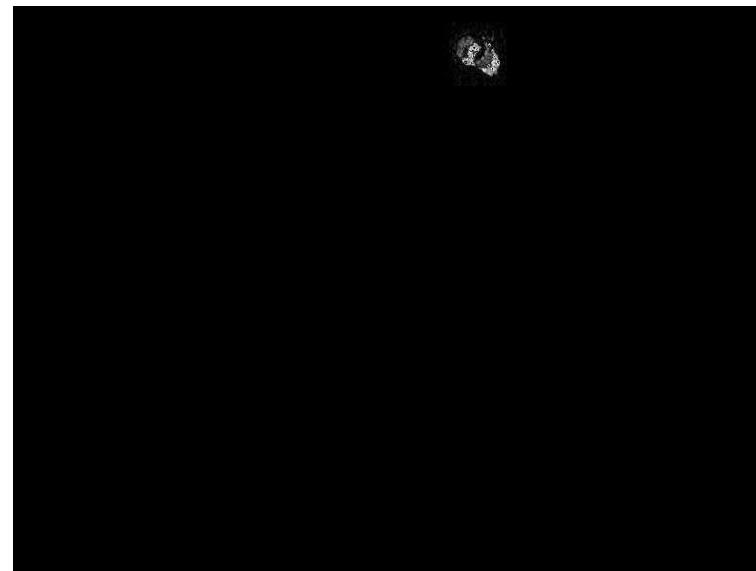
Obraz I_1/I_2 - pozycja początkowa



Obraz I_2/I_1 - pozycja końcowa



Obraz $(I_1/I_2 - I_2/I_1)$
- dynamika ruchu





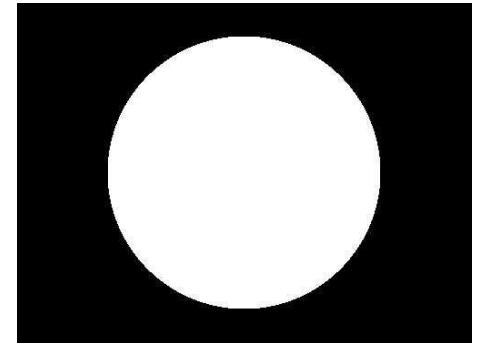
Operacje na bitach obrazów

- najczęściej składanie z obrazami – maskami

00000000 - czarny
11111111 - biały (255)



p	q	$p \wedge q$	p	q	$p \vee q$
0	0	0	0	0	0
0	1	0	0	1	1
1	0	0	1	0	1
1	1	1	1	1	1



operacja AND



operacja OR





Operacja XOR

p	q	$p \oplus q$
0	0	0
0	1	1
1	0	1
1	1	0

- ze względu na fakt bycia własną dwrotną ($\oplus = \oplus^{-1}$, $(X \oplus Y) \oplus Y = X$) była często stosowana do tymczasowego zaznaczania kształtów na obrazach
- usunięcie kształtu odbywa się poprzez jego ponowne narysowanie
- jest to szybka operacja
- algorytm opatentowany !!!: U.S. Patent 4,197,590 firma Cadtrack
- od jakiegoś czasu zniknął z rozwiązań bibliotek GUI :-(

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Version 1.2, November 2002

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